$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

- $\frac{d}{dx}(x^n) = n\cdot x^{n-1}$
- $\boxed{\frac{d}{dx}x^c = c\, x^{c-1}}$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$$

$$\frac{d}{dx}x^{\frac{2}{3}} = \frac{2}{3}\cdot x^{\frac{2}{3}-1} = \frac{2}{3}\cdot x^{-\frac{1}{3}}$$

- $\boxed{\frac{d}{dx}(f\pm g) = \frac{d}{dx}f \pm \frac{d}{dx}g}$, $\boxed{\frac{d}{dx}(c\cdot f(x)) = c\cdot\frac{d}{dx}f}$

$$\frac{d}{dx}(x^3-3\cdot x^{\frac{1}{3}}+2) = \frac{d}{dx}x^3 - \frac{d}{dx}(3\cdot x^{\frac{1}{3}}) + \frac{d}{dx}2$$
$$= 3\cdot x^2 - \underbrace{3\cdot\frac{1}{3}\cdot x^{\frac{1}{3}-1}}_{= x^{-\frac{2}{3}}} + 0 = 3x^2 - x^{-\frac{2}{3}}$$

$f'\cdot g'$

$\neq$

- $\frac{d}{dx}(f\cdot g) = \left(\frac{d}{dx}f\right) g + f\left(\frac{d}{dx}g\right)$
$= f'\cdot g + f\cdot g'$

$$\frac{d}{dx}\cdot\left[(x-2)\cdot(x^2+\sqrt{x})\right] = \left(\frac{d}{dx}(x-2)\right)(x^2+\sqrt{x}) + (x-2)\frac{d}{dx}(x^2+\underset{=\,x^{\frac{1}{2}}}{\sqrt{x}})$$
$$= 1\cdot(x^2+\sqrt{x}) + (x-2)\cdot\left(2\cdot x^{2-1} + \frac{1}{2}x^{\frac{1}{2}-1}\right)$$
$$= x^2+\sqrt{x} + (x-2)\cdot\left(2x+\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$\cdot\ \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{d}{dx}\left(f\cdot\frac{1}{g}\right) = \left(\frac{d}{dx}f\right)\cdot\frac{1}{g} + \underbrace{f\cdot\frac{d}{dx}\left(\frac{1}{g}\right)}_{\substack{\shortparallel \\ f\cdot\left(-\frac{g'}{g^2}\right)}}$$

$$\left(\frac{d}{dx}f\right)\cdot\frac{1}{g} = \frac{f'}{g}$$

$$\boxed{\frac{d}{dx}\left(\frac{1}{g}\right) = -\frac{g'}{g^2}}$$

$$0 = \frac{d}{dx}1 = \frac{d}{dx}\left(g\cdot\frac{1}{g}\right) = g'\cdot\frac{1}{g} + g\cdot\frac{d}{dx}\left(\frac{1}{g}\right)$$

$$g\,\frac{d}{dx}\left(\frac{1}{g}\right) = -\frac{g'}{g} \Rightarrow \boxed{\frac{d}{dx}\left(\frac{1}{g}\right) = -\frac{g'}{g^2}}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g}{g^2} - \frac{f\cdot g'}{g^2} = \boxed{\frac{f'g - f\cdot g'}{g^2} = \frac{d}{dx}\left(\frac{f}{g}\right)}$$

$$\boxed{\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'\cdot g - f\cdot g'}{g^2}} \qquad \boxed{\frac{d}{dx}\frac{1}{g} = -\frac{g'}{g^2}}$$

Ex: $$\frac{d}{dx}\left(\frac{x+1}{x^2+x}\right) = \frac{\left(\frac{d}{dx}(x+1)\right)(x^2+x) - (x+1)\frac{d}{dx}(x^2+x)}{(x^2+x)^2}$$

$$\frac{x+1}{x^2+x} = \frac{1}{x}$$

$$\frac{d}{dx}x^{-1} = -x^{-2}$$

$$= \frac{(1+0)\cdot(x^2+x) - (x+1)\left[2\cdot x^{2-1}+1\right]}{(x^2+x)^2}$$

$$= \frac{\overset{x(x+1)}{\overset{\shortparallel}{(x^2+x)}} - (x+1)\cdot(2x+1)}{(x^2+x)^2}$$

$$= \frac{(x+1)\cdot(-x-1)}{x^2\cdot(x+1)^2} = -\frac{1}{x^2}$$

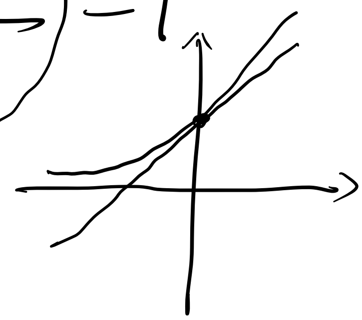
Ex: $$\frac{d}{dx}\left(\frac{P}{Q}\right) = \frac{P'Q - P\cdot Q'}{Q^2}$$

- $$\frac{d}{dx}e^x = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = e^x \cdot \underbrace{\lim_{h\to 0}\frac{e^h-1}{h}}_{=1}$$

$$\frac{e^{x+h}-e^x}{h} = \frac{e^x\cdot e^h - e^x}{h} = \frac{e^x(e^h-1)}{h}$$

$$\lim_{h\to 0}\frac{e^h-1}{h} = \lim_{h\to 0}\frac{e^{0+h}-e^0}{h} = 1$$

$$\frac{f(0+h)-f(0)}{h}$$



$$\frac{d}{dx}e^x = e^x\cdot 1 = \boxed{e^x = \frac{d}{dx}e^x}$$

$$\frac{d}{dx}(x^2\cdot e^x) = 2x\cdot e^x + x^2\cdot e^x = (2x+x^2)e^x$$

$$f\cdot g \qquad \underbrace{\frac{d}{dx}(x^2)\cdot e^x} \qquad x^2\cdot\frac{d}{dx}e^x$$

$$\frac{d}{dx}\left(\frac{e^x}{x+1}\right) = \frac{\left(\frac{d}{dx}e^x\right)(x+1) - e^x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{e^x(x+1) - e^x\cdot 1}{(x+1)^2} = \frac{e^x\cdot x}{(x+1)^2}$$

$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \cos x.$$

$$\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cdot \cosh + \cos x \cdot \sinh - \sin x}{h}$$

$$= \cos x \cdot \frac{\sinh}{h} + \frac{\sin x \cdot (1 - \cosh)}{h}$$

$$\downarrow h \to 0 \qquad\qquad \downarrow h \to 0$$

$$\cos x \cdot 1 + \sin x \cdot 0$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = - \sin x$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\left(\frac{d}{dx}\sin x\right)\cos x - \sin x \cdot \frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin x \cdot (+\sin x)}{\cos^2 x} = \frac{\overbrace{\cos^2 x + \sin^2 x}^{1}}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2 = \sec^2 x = (\sec x)^2$$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\frac{\cos x}{\sin x} = - \left(\frac{1}{\sin^2 x}\right) = - \csc^2 x$$

$$\frac{d}{dx}\left(\underbrace{\cos^2 x}_{=\cos x \cdot \cos x}\right) = \underbrace{\frac{d}{dx}(\cos x)}_{=-\sin x} \cdot \cos x + \cos x \cdot \frac{d}{dx}(\cos x)$$

$$= -\sin x \cdot \cos x + \cos x \cdot (-\sin x)$$

$$= -2 \cdot \sin x \cdot \cos x$$

Ex: $$\frac{d}{dx}\left(\frac{x \cdot \sin x}{2\cos x + 1}\right) = \frac{\frac{d}{dx}(x\sin x) \cdot (2\cos x + 1) - (x\sin x)\frac{d}{dx}(2\cos x + 1)}{(2\cos x + 1)^2}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f' \cdot g - f \cdot g'}{g^2}, \quad \frac{d}{dx}(fg) = f'g + fg'$$

$$= \frac{(1\cdot\sin x + x\cdot\cos x)\cdot(2\cos x+1) - (x\sin x)\cdot(-2\cdot\sin x+0)}{(2\cos x+1)^2}$$

$$\frac{d}{dx} x^c = c\cdot x^{c-1}$$

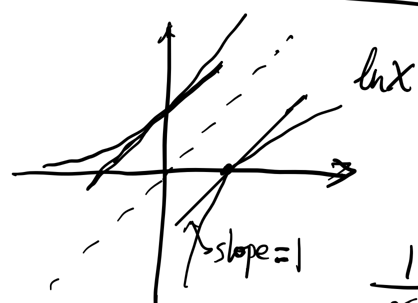
$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}(f \pm g) = f' \pm g' \qquad \frac{d}{dx}(f\cdot g) = f'\cdot g + f\cdot g'$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'\cdot g - f\cdot g'}{g^2}$$

$$\frac{d}{dx}(\underset{\substack{\| \\ \log_e x}}{\ln x}) = \lim_{h\to 0}\frac{\ln(x+h)-\ln x}{h}$$



$$\ln\frac{x+h}{x} = \ln(1+\underset{\substack{\| \\ t}}{\frac{h}{x}})$$

$$= \lim_{h\to 0}\frac{\ln\frac{x+h}{x}}{h}$$

$$\frac{1}{x} \;\; \|$$

$$\boxed{\ln b - \ln a = \ln\frac{b}{a}} \quad = \lim_{t\to 0}\frac{\ln(1+t)}{t\cdot x} = \frac{1}{x}\cdot\underbrace{\lim_{h\to 0}\frac{\ln(1+t)-\ln 1}{t}}_{\text{derivative of ln at 1}} = \frac{1}{x}\cdot 1$$

$$f'(a) = \lim_{t \to 0} \frac{f(a+t) - f(a)}{t}$$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}$$