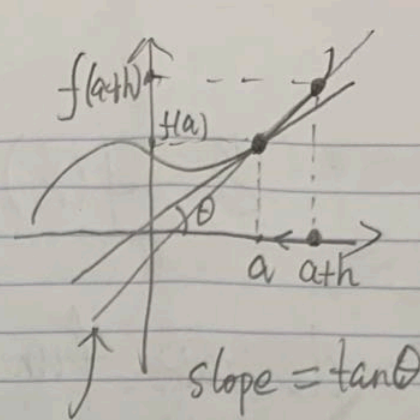
$f(x)$



$f'(a)$ $\quad$ slope of this secant line is

$$\text{slope of the tangent line at } (a, f(a)) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \frac{f(a+h)-f(a)}{a+h-a}$$

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}} \leftarrow \text{instantaneous change rate}$$

Ex: $\boxed{f(x) = x^2}$

$$f'(a) = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} = \lim_{h \to 0} \frac{2ah+h^2}{h}$$

$$\frac{(a+h)^2 - a^2}{h}: \quad (a+h)^2 - a^2 = \cancel{a^2}+2ah+h^2-\cancel{a^2}$$

derivative fct. of f

$$\boxed{f'(x) = 2x} \Leftarrow \boxed{f'(a) = 2a} = \lim_{h \to 0} (2a+h)$$

If f represents the distance function and x is the time variable, then $f'(a) = \underbrace{\text{velocity}}_{\text{instantaneous}}$ at time $x = a$.

$y = f(x)$

derivative function of f:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} \rightsquigarrow \underline{f'(a)}$$

Ex: $f(x) = \sqrt{x}$

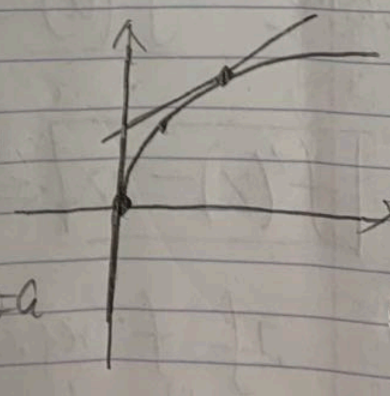
$$\cancel{x} + h - \cancel{x}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\frac{d}{dx}\sqrt{x} = f'(x) = \frac{1}{2\sqrt{x}} = \lim_{h \to 0} \left(\frac{1}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$f'(x) = \frac{d}{dx} f = \frac{df}{dx}$$

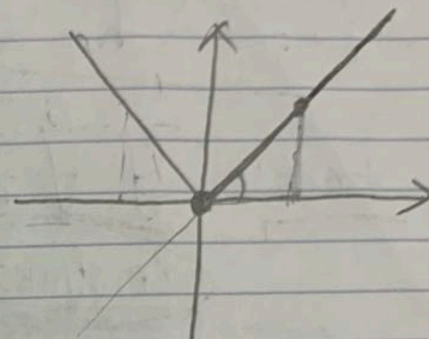
$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}} \quad x \neq 0$$



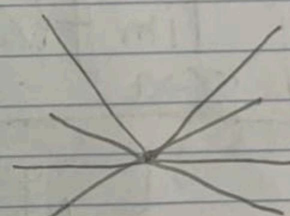
Fact: If the tangent line is vertical at $x = a$ then $f'(a)$ DNE.

Ex: $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$$f'(x) = \begin{cases} 1 & x > 0 \\ \text{DNE} & x = 0 \\ -1 & x < 0 \end{cases}$$



$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$



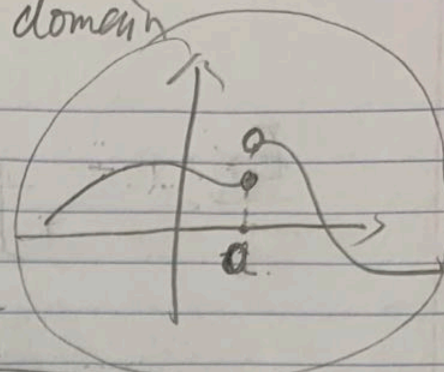
$$= \lim_{h \to 0} \frac{|h|}{h} = \text{DNE}$$

$$\lim_{h \to 0^+} \frac{|h|}{h} = 1, \quad \lim_{h \to 0^-} \frac{\overset{=-h}{|h|}}{h} = -1$$

Fact: If $\underbrace{}_{\text{graph of}}$ f has a sharp corner at $x = a$, then $f'(a)$ DNE.

Def: f is $\underbrace{\text{differentiable}}_{\text{smooth}}$ at $x = a$ if $f'(a)$ exist.

To get $f'(a)$, need $\underset{a}{\overset{x}{\shortparallel}}$ in the domain



$$\underset{=}{f'(a)}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a) \text{ exist}$$

$\Updownarrow$

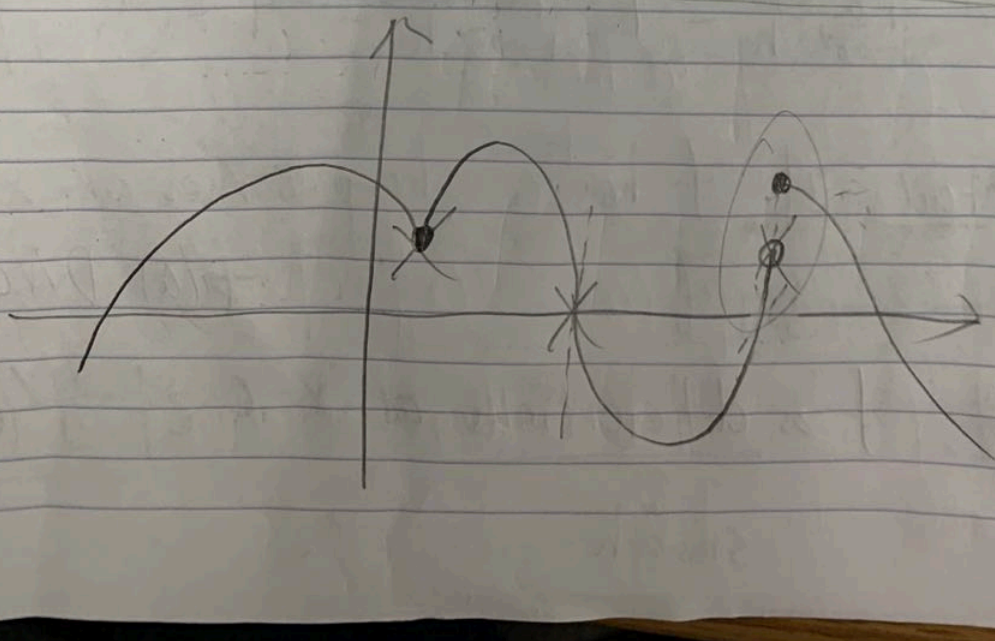
<u>Fact</u>: If f is differentiable at $x=a$, then <u>f is</u> continuous at $x=a$.

$$\boxed{\lim_{x \to a} f(x) \overset{?}{=} f(a)}$$

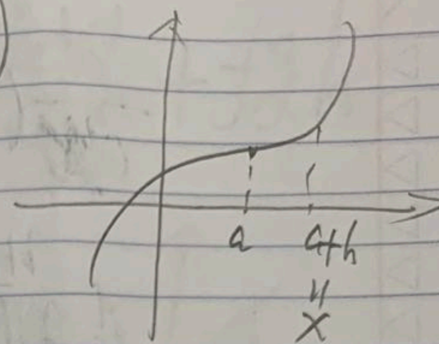
$\Downarrow$?

<u>Q</u>: If $\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)}$ exist

then why is $\boxed{\lim_{x \to a} f(x) = f(a)}$?



$$f'(a) = \lim_{h\to 0} \frac{f(\overset{x}{\overset{\parallel}{a+h}})-f(a)}{h}$$



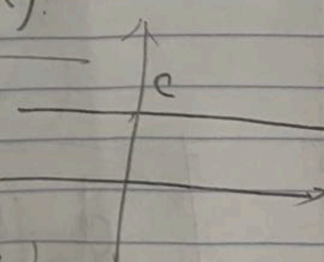
$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{y\to a} \frac{f(y)-f(a)}{y-a}$$

$$\lim_{x\to a} f(x) = \left(\frac{f(x)-f(a)}{x-a}\right)\cdot(x-a) + f(a)$$

$$= f(a).$$

- $(f \pm g)'(x) = f'(x) \pm g'(x)$



- $\boxed{\frac{d}{dx} c = 0}$
- $\frac{d}{dx}(c\cdot f(x)) = \lim_{h\to 0} \frac{c\cdot f(x+h) - c\cdot f(x)}{h}$

$$\parallel$$

$$c\frac{d}{dx}f(x) = c\cdot \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) \cancel{=} f' \cdot g'$$

$$\lim_{h\to 0} \frac{(f\cdot g)(x+h) - (f\cdot g)(x)}{h} = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x+h)\cdot g(x)}{h} + \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot g(x)$$

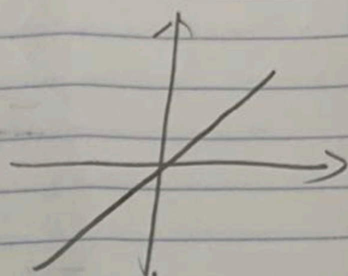
$$= f(x) \cdot g'(x) + f'(x)\cdot g(x)$$

$$\boxed{\frac{d}{dx}(f\cdot g) = f'\cdot g + f\cdot g'}$$

product rule (Leibniz rule)

$$\frac{d}{dx}(f\cdot g\cdot h) = f'gh + fg'h + fgh'$$

$$\frac{d}{dx}(x) = 1$$



$$1\cdot x^{n-1}$$

$$\frac{d}{dx}(x^n) = \left(\frac{d}{dx}x\right)\cdot x\cdots x + x\cdot\left(\frac{d}{dx}x\right)\cdots x \underbrace{+\cdots\cdots}_{n}$$

$$x^n = \underbrace{x\cdots x}_{n}$$

$$\boxed{\frac{d}{dx}x^n = n\cdot x^{n-1}}$$

$$\boxed{\frac{d}{dx}\left(\frac{1}{x}\right) = (-1)\cdot x^{-2} = -\frac{1}{x^2}}$$

$$\frac{1}{x} = x^{-1}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = -\frac{1}{x^2}$$

$$\boxed{\frac{d}{dx}(x^c) = c\cdot x^{c-1} \quad \text{for any } c}$$

Ex: $\frac{d}{dx}(x^{10}-2\cdot x^2+3)$

$$= \frac{d}{dx}x^{10} - \frac{d}{dx}(2x^2) + \frac{d}{dx}3$$

$$= 10\cdot x^9 - 2\underbrace{\frac{d}{dx}x^2}_{=2x} + 0$$

$$= 10\cdot x^9 - 4x$$