(a) $\lim_{x\to 0} g(x) = 3$ $\lim_{x\to 7} g(x) = DNE$

$\lim_{x\to 2^-} g(x) = -\infty$

$\lim_{x\to 5^-} g(x) = -3$

$\lim_{x\to 5^+} g(x) = 2$

(a) $$\lim_{x\to 3} \frac{x-3}{10-\sqrt{x+97}} = \lim_{x\to 3} \frac{(x-3)\cdot(10+\sqrt{x+97})}{(10-\sqrt{x+97})(10+\sqrt{x+97})} = \frac{(x-3)(10+\sqrt{x+97})}{\underset{\substack{\parallel \\ 3-x}}{100-(x+97)}}$$

$$= \lim_{x\to 3} \ominus(10+\sqrt{x+97})$$

$$= -(10+\sqrt{100}) = -(10+10) = -20$$

(b) $$\frac{36-x^2}{\frac{1}{x}-\frac{1}{6}} = \frac{(6+x)\cancel{(6-x)}}{\frac{\cancel{6-x}}{6x}} = (6+x)\cdot 6x$$

$$\lim_{x\to 6} \frac{36-x^2}{\frac{1}{x}-\frac{1}{6}} = \lim_{x\to 6}(6+x)\,6x = (6+6)\cdot 6\times 6 = 12\times 6\times 6 = 72\times 6 \underset{\parallel}{} 432$$

(c) $$\lim_{x\to 0} \frac{x^2\cdot \csc(3x)}{\cos(7x)\sin(4x)} = \frac{x^2\cdot\frac{1}{\sin(3x)}}{\cos(7x)\cdot\sin(4x)} = \frac{x^2}{\cos(7x)\cdot\sin(3x)\cdot\sin(4x)}$$

$$\frac{\frac{1}{12}}{\lim_{x\to 0}\cos(7x)\,\lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\lim_{x\to 0}\frac{\sin(4x)}{4x}} = \lim_{x\to 0}\frac{\frac{1}{12}}{\cos(7x)\,\frac{\sin(3x)}{3x}\,\frac{\sin(4x)}{4x}} = \frac{\frac{x^2}{3x\cdot 4x}}{\cos(7x)\cdot\frac{\sin(3x)}{3x}\cdot\frac{\sin(4x)}{4x}}$$

$$\parallel$$

$$\frac{\frac{1}{12}}{\cos(0)\cdot 1\cdot 1} = \frac{1}{12}$$

$$\boxed{\lim_{x\to 0}\frac{\sin(k\cdot x)}{k\cdot x} = 1}$$

(d) $$\lim_{x\to 2^-}\frac{6x^2-7x}{x^2-4}=\lim_{x\to 2^-}\frac{x(6x-7)}{(x+2)(x-2)}=\frac{2\cdot(12-7)}{4\cdot 0^-}=\frac{10}{0^-}$$
$$\frac{10}{0^-} = -\infty$$

$$f(x)=\begin{cases}\dfrac{(x+1)^2-16}{2x-6} & x<3\\ 3-\ln(x-2) & x\geq 3\end{cases}=\begin{cases}\dfrac{x+5}{2} & x<3\\ 3-\ln(x-2) & x\geq 3\end{cases}$$

Continuous at a: $\boxed{\lim_{x\to a} f(x)=f(a).}$

$$\frac{(x+1)^2-16}{2x-6}=\frac{\overset{(x+5)}{(x+1+4)}\,\overset{(x-3)}{(x+1-4)}}{2(x-3)}=\frac{x+5}{2}\quad x<3$$

$f(x)$ is continuous when $x<3$ or $(-\infty,3)$

when $x>3$ or $(3,+\infty)$.

at $x=3$: Check $\lim_{x\to 3} f(x)$ exists or not

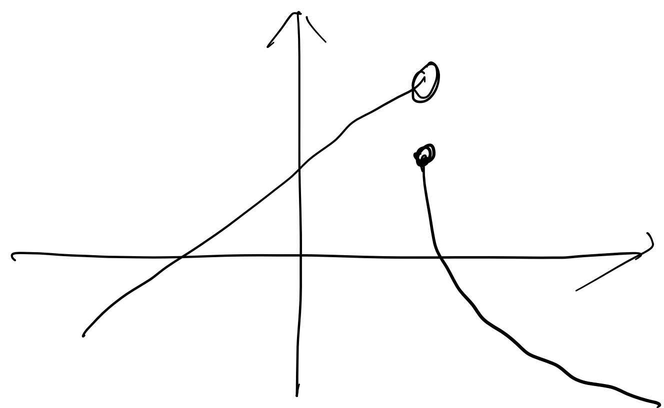
$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\frac{x+5}{2}=\frac{3+5}{2}=4$$

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(3-\ln(x-2))=3-\ln(3-2)=3$$

$\lim\limits_{x\to 3} f(x)$ DNE $\Rightarrow$ $f(x)$ is not continuous at $x=3$.

$\Rightarrow$ f is continuous on $(-\infty, 3) \cup (3, +\infty)$

$(-\infty, 3)$, $(3, +\infty)$



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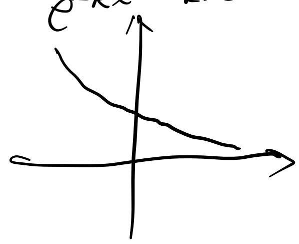
horizontal asymptotes

$\lim\limits_{x\to+\infty} f(x)$

$\lim\limits_{x\to-\infty} f(x)$

$e^{-k\cdot x}$, $k>0$

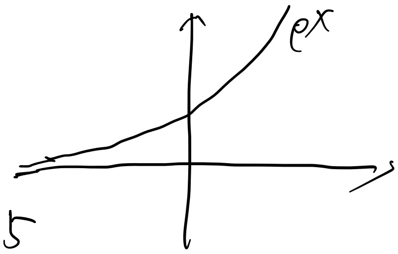


$$\lim_{x\to+\infty} \frac{2e^{x}-15}{5e^{3x}+8} = \frac{\frac{2e^{x}-15}{e^{3x}}}{\frac{5e^{3x}+8}{e^{3x}}} = \lim_{x\to+\infty} \frac{2\cdot e^{-2x}-15\cdot e^{-3x}}{5+8\cdot e^{-3x}}$$

$$= \frac{2\cdot 0 - 15\cdot 0}{5+8\cdot 0} = 0$$

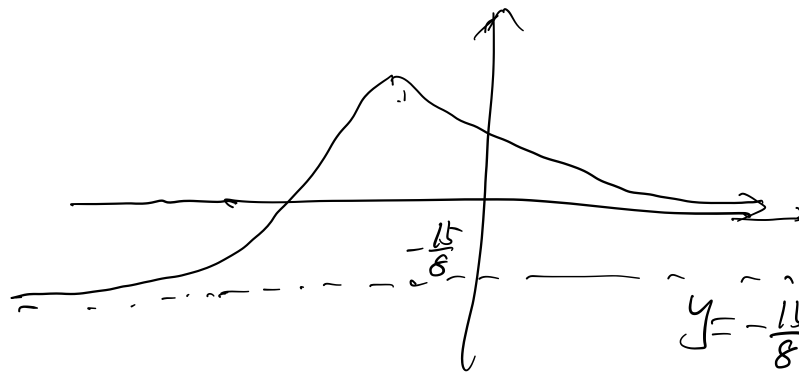
$$\lim_{x\to-\infty} \frac{2e^x-15}{5e^{3x}+8} = \frac{2\cdot e^{-\infty}-15}{5\cdot(e^{-\infty})^3+8}$$

$e^{3x} = (e^x)^3$



$$= \frac{0-15}{5\cdot 0+8} = -\frac{15}{8}$$

$\Rightarrow$ $y=0$ and $y=-\frac{15}{8}$ are horizontal asymptotes



$$f(x) = \frac{x^2-8x+12}{3x^2-8x+4}$$

$\begin{matrix} 3 & -2 \\ 1 & -2 \end{matrix}$

Find <u>Domain</u>: $3x^2-8x+4 = (3x-2)(x-2)=0$

$\Rightarrow x = \frac{2}{3}$ or 2

$\Rightarrow$ $\boxed{\text{Domain } (-\infty, \tfrac{2}{3}) \cup (\tfrac{2}{3}, 2) \cup (2, +\infty)}$

$$f(x) = \frac{x-6}{3x-2} = \frac{x^2-8x+12}{3x^2-8x+4}$$

$x^2-8x+12=(x-6)(x-2)$

vertical asymptotes: $\lim_{x\to a^{\mp}} f(x) = \pm\infty$

possible vertical asymptotes: $x=\frac{2}{3}$, $x=2$.

$$\lim_{x\to\left(\frac{2}{3}\right)^-} \frac{x^2-8x+12}{3x^2-8x+4} = \lim \frac{(x-6)\cancel{(x-2)}}{(3x-2)\cdot\cancel{(x-2)}}$$

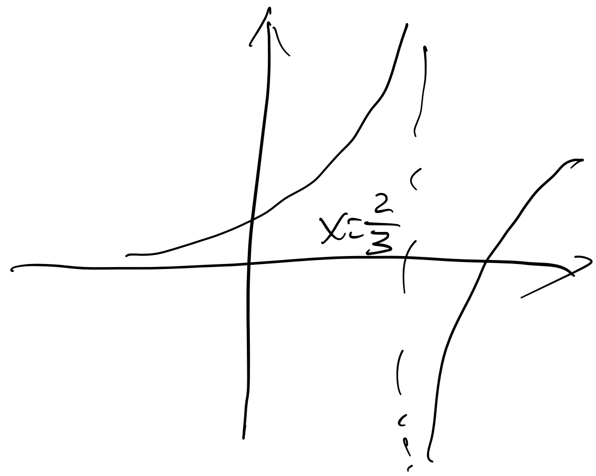
$$= \lim_{x\to\left(\frac{2}{3}\right)^-} \frac{x-6}{3x-2} = \frac{\frac{2}{3}-6}{0^-} = +\infty$$

$$\lim_{x\to\left(\frac{2}{3}\right)^+} f(x) = \frac{\frac{2}{3}-6}{0^+} = -\infty$$

$$\lim_{x\to 2} \frac{x^2-8x+12}{3x^2-8x+4} = \lim_{x\to 2} \frac{x-6}{3x-2} = \frac{2-6}{3\cdot 2-2} = -1 \neq \pm\infty$$

$$\frac{x^2-8x+12}{3x^2-8x+4} = \frac{(x-6)\cancel{(x-2)}}{(3x-2)\cancel{(x-2)}}$$

$\Rightarrow$ The only vertical asymptote is $x = \frac{2}{3}$



horizontal asymptotes:

$$\lim_{x \to +\infty} \frac{x^2-8x+12}{3x^2-8x+4} = \lim_{x\to+\infty} \frac{\frac{x^2-8x+12}{x^2}}{\frac{3x^2-8x+4}{x^2}}$$

$$\frac{1}{3} = \frac{1-0+0}{3-0+0} = \lim_{x\to+\infty} \frac{1-\frac{8}{x}+\frac{12}{x^2}}{3-\frac{8}{x}+\frac{4}{x^2}}$$

$$\lim_{x\to-\infty} \frac{x^2-8x+12}{3x^2-8x+4} = \frac{1}{3}$$

$\Rightarrow$ $y = \frac{1}{3}$ is the only horizontal asymptote

Rmk: $\dfrac{x^3-2x+8}{\sqrt{x^6-x}} = f(x)$

$\cancel{\lim_{x\to+\infty} f(x)}$

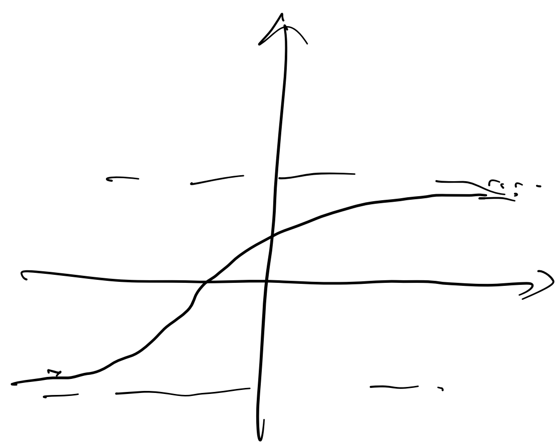
$$\lim_{x\to-\infty} f(x) = \frac{\dfrac{x^3-2x+8}{x^3}}{\dfrac{\sqrt{x^6-x}}{x^3}} = \frac{1-\dfrac{2}{x^2}+\dfrac{8}{x^3}}{-\sqrt{1-\dfrac{1}{x^5}}}$$

$$x^3 = -\sqrt{x^6}, \quad \sqrt{x^6} = |x^3|$$

$$\Big\downarrow x\to-\infty$$

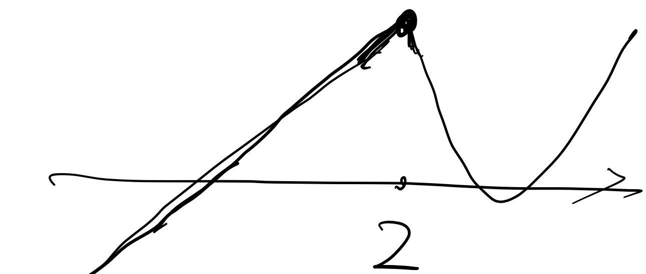
$$\frac{1-0+0}{-\sqrt{1-0}} = -1$$

$$\lim_{x\to+\infty} f(x) = 1$$



$$f(x) = \begin{cases} Ax+B-4 & x<2 \\ 9 & x=2 \\ Ax^2-5 & x>2 \end{cases}$$

Continuous at $x=2$: $\boxed{\lim_{x\to 2} f(x) = f(2) = 9}$



$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (Ax+B-4) = 2A+B-4 = 9$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (Ax^2-5) = A\cdot 2^2-5 = 4A-5 = 9$$

$$\Rightarrow A = \frac{9+5}{4} = \frac{14}{4} = \frac{7}{2} = 3.5$$

$$B = 9+4-2A = 13-2\times\frac{7}{2} = 6$$