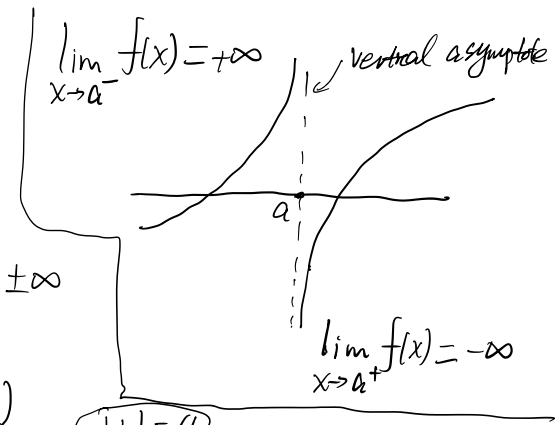
Ex: $\lim_{x\to -1} \frac{x^2-2x+3}{x^2-4x-5}$

$= \frac{(-1)^2-2\cdot(-1)+3}{(-1)^2-4\cdot(-1)-5} = \frac{1+2+3}{1+4-5} = \frac{6}{0} \neq \pm\infty$

$x^2-4x-5 = (x-(-1))\cdot(x-5)$



$-1+b=4 \Rightarrow b=5$

$(-1)\cdot b = -5$

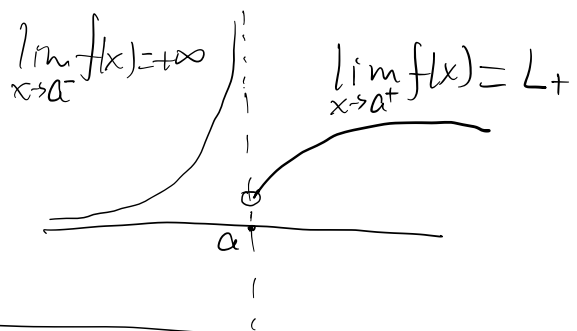
- $\lim_{x\to(-1)^-} \frac{x^2-2x+3}{(x-(-1))(x-5)} \to \frac{6}{0^-\cdot(-1-5)} = \frac{6}{\ominus 6\cdot 0^\ominus} = \frac{6}{6\cdot 0^+} = \frac{6}{0^+} = +\infty$

  $0^\ominus \quad \ominus 6$

- $\lim_{x\to(-1)^+} \frac{x^2-2x+3}{(x-(-1))(x-5)} \to 6 = \frac{6}{0^\oplus(\ominus 6)} = \frac{6}{0^-} = -\infty$

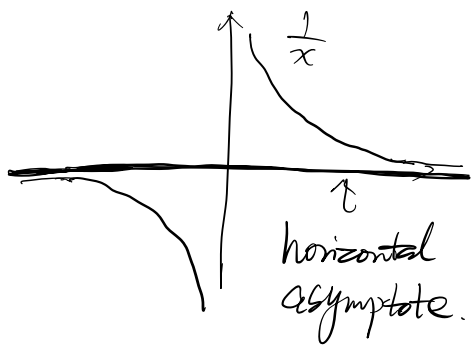
  $0^+ \quad -6$

Ex:



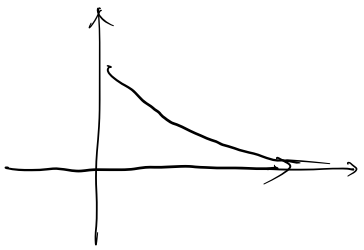
Ex: $\lim_{x\to+\infty} \frac{1}{x} = \frac{1}{+\infty} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = \frac{1}{-\infty} = 0$



$a>0$

$$\lim_{x\to+\infty}\frac{1}{x^a}=\frac{1}{+\infty}=0 \qquad (x^{-a})$$



$$\lim_{x\to+\infty}\frac{1}{\sqrt{x}}=\frac{1}{\sqrt{+\infty}}=\frac{1}{+\infty}=0 \qquad (\sqrt{x}=x^{\frac{1}{2}})$$

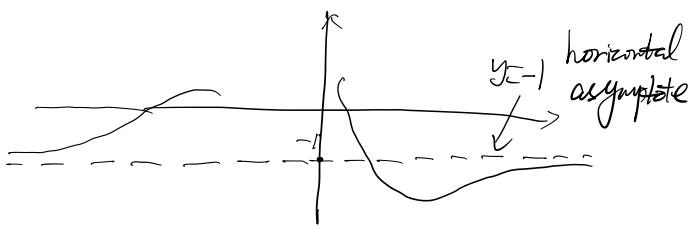
$$\lim_{x\to+\infty}\frac{2x-3}{x-100}=\frac{2\cdot\infty-3}{\infty-100}=\frac{+\infty}{+\infty}$$

$$\frac{\frac{2x-3}{x}}{\frac{x-100}{x}}=\lim_{x\to+\infty}\frac{2-\frac{3}{x}}{1-\frac{100}{x}}=\frac{2-\lim_{x\to+\infty}\frac{3}{x}}{1-\lim_{x\to+\infty}\frac{100}{x}}=\frac{2-0}{1-0}=2$$

$$\lim_{x\to+\infty}\frac{x^3-3x}{-x^3+x^2-4}=\frac{\frac{x^3-3x}{x^3}}{\frac{-x^3+x^2-4}{x^3}}=\lim_{x\to+\infty}\frac{1-\frac{3}{x^2}}{-1+\frac{1}{x}-\frac{4}{x^3}}$$

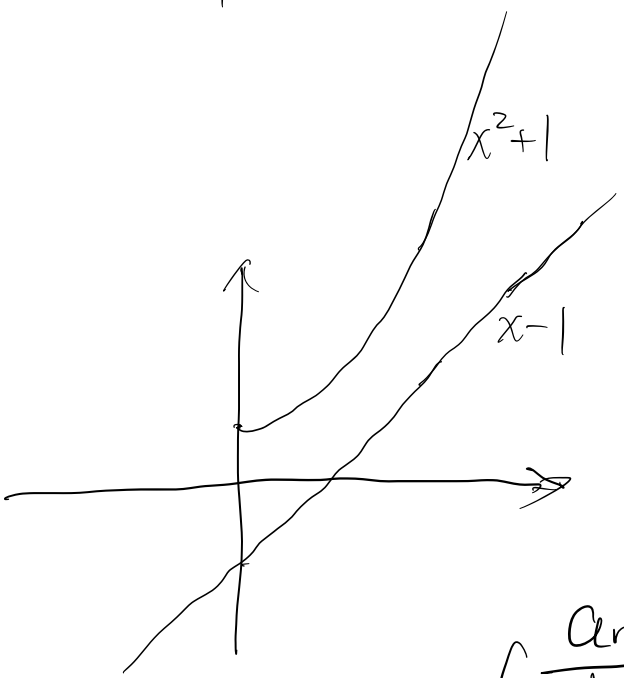
$$\frac{1}{-1}=\frac{1-0}{-1+0-0}=\frac{\lim_{x\to+\infty}1-\lim_{x\to+\infty}\frac{3}{x^2}}{-1+\lim_{x\to+\infty}\frac{1}{x}-\lim_{x\to+\infty}\frac{4}{x^3}}$$

$$\lim_{x\to-\infty}\frac{x^3-3x}{-x^3+x^2-4}=\lim_{x\to-\infty}\frac{1-\frac{3}{x^2}}{-1+\frac{1}{x}-\frac{4}{x^3}}=\frac{1-0}{-1+0-0}=-1$$



- $\lim_{x\to+\infty} \dfrac{x^2-1}{x^2+1} = \dfrac{\frac{x-1}{x^2}}{\frac{x^2+1}{x^2}} = \lim_{x\to+\infty} \dfrac{\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x^2}}$

$$0 = \frac{0-0}{1-0} = \frac{\lim_{x\to+\infty}\frac{1}{x} - \lim_{x\to+\infty}\frac{1}{x^2}}{\lim_{x\to+\infty} 1 - \lim_{x\to+\infty}\frac{1}{x^2}}$$



$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = \begin{cases} \dfrac{a_m}{b_n} & \deg P = \deg Q \\ 0 & \deg P < \deg Q \\ +\text{or}-\infty & \deg P > \deg Q \end{cases}$$

(m = $\deg P$, n = $\deg Q$)

$P(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_0, \quad a_m \neq 0$

$Q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_0, \quad b_n \neq 0$

$$\lim_{x\to+\infty} \frac{x^2+1}{x-1} = \frac{\frac{x^2+1}{x}}{\frac{x-1}{x}} = \lim_{x\to+\infty} \frac{x+\frac{1}{x}}{1-\frac{1}{x}}$$

$$+\infty = \frac{+\infty}{1} = \frac{+\infty+0}{1-0} = \frac{+\infty+\frac{1}{+\infty}}{1-\frac{1}{+\infty}}$$

- $+\infty \pm a = +\infty$
- $+\infty \cdot +\infty = +\infty$
- $+\infty \cdot -\infty = -\infty$

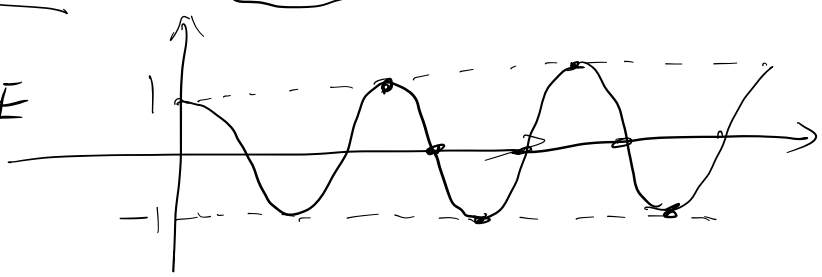
$+\infty - (+\infty)$, $\frac{+\infty}{+\infty}$ — give no information

$$\lim_{x\to-\infty} \frac{x^4+1}{x-100} = \frac{\frac{x^4+1}{x}}{\frac{x-100}{x}} = \lim_{x\to-\infty} \frac{x^3+\frac{1}{x}}{1-\frac{100}{x}}$$

$$-\infty = \frac{-\infty}{1} = \frac{-\infty+0}{1-0} = \frac{(-\infty)^3+\frac{1}{-\infty}}{1-\frac{100}{-\infty}}$$

- $\lim_{x\to+\infty} \frac{\cos(x^3)}{\sqrt{x}} = \frac{\cos((+\infty)^3)}{\sqrt{+\infty}} = \frac{\cos(+\infty)}{+\infty}$ DNE

$\lim_{x\to+\infty} \cos(x)$ DNE



$$\frac{-1}{\sqrt{x}} \le \frac{\cos(x^3)}{\sqrt{x}} \le \frac{1}{\sqrt{x}} \quad \Rightarrow \lim_{x\to+\infty} \frac{\cos(x^3)}{\sqrt{x}} = 0$$

$x \to +\infty$ $\quad$ $x \to +\infty$ $\quad$ $x \to +\infty$

$\frac{-1}{\sqrt{+\infty}} = \frac{-1}{+\infty} = 0^-$ $\quad$ 0 $\quad$ $\frac{1}{+\infty} = 0$

$$\lim_{x\to 0} x\cos\left(\frac{1}{x}\right) = 0 \cdot \cos\left(\frac{1}{0}\right)$$

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$

$$\Downarrow$$

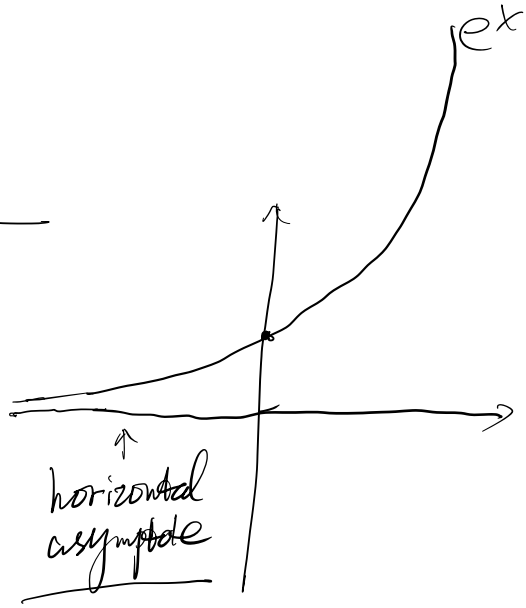
$$-|x| \le x\cos\left(\frac{1}{x}\right) \le |x|$$

$x \to 0$ $\quad$ $\downarrow$ $\quad$ $x \to 0$

0 $\quad$ 0 $\quad$ 0

$$\lim_{x\to+\infty} e^x = +\infty$$

$$\lim_{x\to-\infty} e^x = e^{-\infty} = 0$$

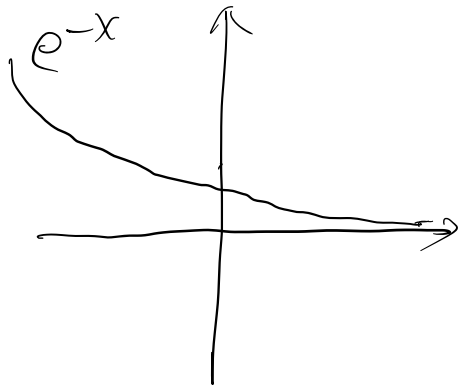


exponential growth

$>>$ polynomial growth

$$\lim_{x\to+\infty} e^{-x} = e^{-\infty} = 0$$

$$\lim_{x\to+\infty} e^{-x} = e^{-(-\infty)} = e^{+\infty} = +\infty$$



- $\lim_{x\to+\infty} \dfrac{2\cdot e^x + 1}{1\cdot e^x - 1} = \dfrac{+\infty}{+\infty}$ ✗

$$\frac{\dfrac{2\cdot e^x + 1}{e^x}}{\dfrac{e^x - 1}{e^x}} = \frac{2 + \dfrac{1}{e^x}}{1 - \dfrac{1}{e^x}} \xrightarrow{x\to+\infty} \frac{2 + \dfrac{1}{+\infty}}{1 - \dfrac{1}{+\infty}} = \frac{2+0}{1-0} = 2$$