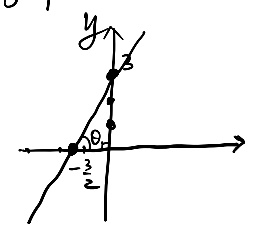
Linear functions $\longrightarrow$ graph are lines.

$f(x) = 2x + 3$

$\uparrow$ slope $\qquad \uparrow$ y-intercept



$0 = f(x) = 2x+3 \Rightarrow 2x = -3$

$\Rightarrow x = -\frac{3}{2}$ x-intercept.

$2 = \tan\theta = \frac{3}{\frac{3}{2}} = 2$

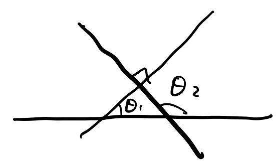
- $f(x) = \underset{\text{slope}}{k}\,x + b$ $\leftarrow$ y-intercept

  x-intercept $= -\frac{b}{k}$

  $k\cdot x + b = 0 \Rightarrow$

- $l_1 \,//\, l_2 \iff k_1 = k_2$

  slopes: k_1 $\qquad k_2$



$l_1 \perp l_2 \iff \theta_2 = \theta_1 + \frac{\pi}{2}$

$$\tan(\theta_2) = \tan\left(\theta_1 + \frac{\pi}{2}\right) = \frac{\sin(\theta_1 + \frac{\pi}{2})}{\cos(\theta_1 + \frac{\pi}{2})} = \frac{\cos(\theta_1)}{-\sin(\theta_1)} = -\frac{1}{\tan\theta_1}$$

$\Updownarrow$

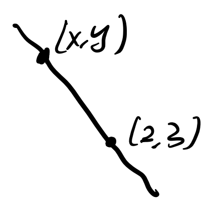
$k_1 \cdot k_2 = -1$

Example: Find the equation for the line passing through $(2,3)$, which perpendicular to line with slope $\frac{1}{2}$

sol: $k = \frac{-1}{\frac{1}{2}} = -2. = \frac{y-3}{x-2}$

$\Rightarrow y - 3 = -2(x-2) \iff \boxed{y = -2x + 7}$

$\qquad\qquad\quad \parallel$
$\qquad\qquad\; -2x+4$



- $(x_0, y_0) \in l$, slope k $\Longrightarrow y - y_0 = k\cdot(x - x_0)$

  $\boxed{y = k(x - x_0) + y_0}$

• $(x_1,y_1)\in l$, $(x_2,y_2)\in l$ $\Rightarrow k=\frac{y_2-y_1}{x_2-x_1} \Rightarrow \boxed{y=\frac{y_2-y_1}{x_2-x_1}(x-x_1)+y_1}$

• Quadratic function

Graph

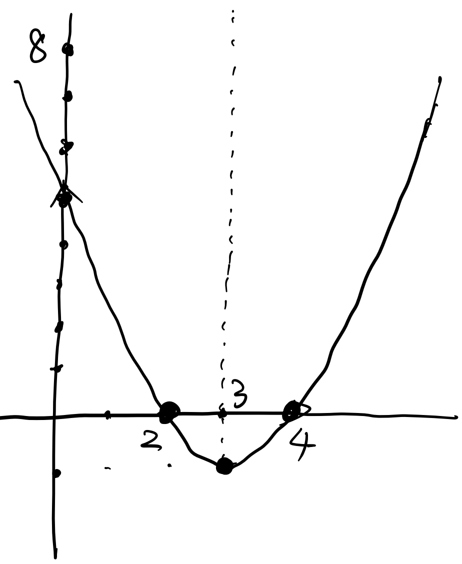
$$f(x)=a\cdot x^2+b\cdot x+C \quad (a\neq 0) \rightsquigarrow \text{parabola}.$$

$$\boxed{x^2+2k\,x+k^2=(x+k)^2}$$

Ex: $f(x)=x^2-6x+8$

$$=x^2-6x+\underbrace{\left(\frac{-6}{2}\right)^2}_{9}-1$$

$$=(x-3)^2-1\geq -1$$



$0=x^2-6x+8=(x-2)\cdot(x-4)\Rightarrow x=2$ or 4

$f(x)>0 \Leftrightarrow x\in(-\infty,2)\cup(4,+\infty)$

$f(x)<0 \Leftrightarrow x\in(2,4)$

$$x^2+\underbrace{(-6)}_{=2(-3)=2\cdot k}x+8=\left(x^2-6x+(-3)^2\right)-(-3)^2+8$$

$$=(x-3)^2-9+8=(x-3)^2-1$$

Ex: $x^2+4x-9 = x^2+4x+2^2-4-9$

$4 = 2\cdot 2$

$= (x+2)^2-13$

$$\boxed{0 = a\cdot x^2+bx+c \Rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$$\parallel \qquad\qquad \Uparrow$$

$$a\cdot\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right)-\frac{b^2}{4a}+c=0$$

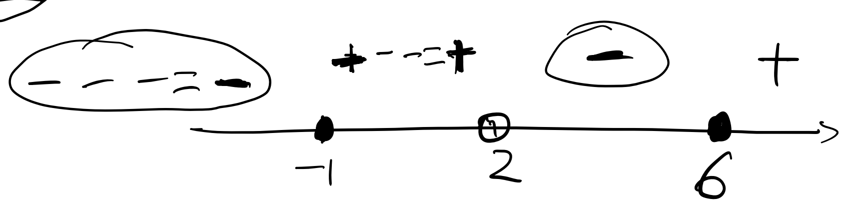
- Ex: solve $\dfrac{x^2-5x-6}{x-2}\le 0$

sol: $\dfrac{(x-6)\cdot(x+1)}{x-2}\le 0 \iff \boxed{\begin{array}{l} x\neq 2 \\ (x-6)\cdot(x+1)\cdot(x-2)\le 0\end{array}}$

$(x-6)\cdot(x+1)\cdot(x-2) = \dfrac{(x-6)\cdot(x+1)}{(x-2)}\cdot(x-2)^2$

$(x-(-1))\cdot(x-2)(x-6)$

$= (x+1)\cdot(x-2)(x-6)$.



$\boxed{\text{Solution set:}}$ $(-\infty,-1]\cup(2,6]$

- Ex: solve $\frac{2x+5}{2} - \frac{3x}{x-2} > x$ $\quad x \neq 2$

Multiply $2\cdot(x-2)$

case 1: $x-2>0$: $\underbrace{(2x+5)(x-2) - 3x\cdot 2}_{} > \underbrace{x\cdot 2(x-2)}_{}$

$\Updownarrow$

$x>2$

$\cancel{2x^2} - \cancel{4x} + 5x - 10 - 6x > \cancel{2x^2} - \cancel{4x}$

$\parallel$

$-x-10>0 \Leftrightarrow x<-10$

case 2: $x-2<0$: $(2x+5)(x-2) - 3x\cdot 2 < x\cdot 2(x-2)$

$\Updownarrow$

$x<2$

$\cdots \Leftrightarrow x>-10$

$\Rightarrow$ $(-10, 2)$ is solution set.

$$\frac{2x+5}{2} - \frac{3x}{x-2} - x > 0$$

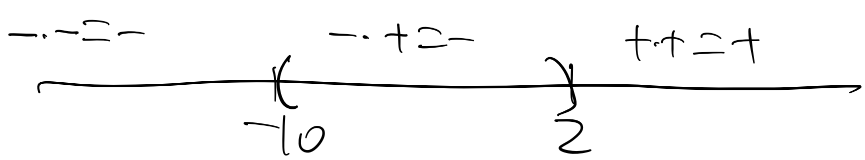
$\parallel$

$$\frac{(2x+5)(x-2) - 6x - 2x(x-2)}{2(x-2)} = \frac{-x-10}{2(x-2)} > 0$$

$\Updownarrow$

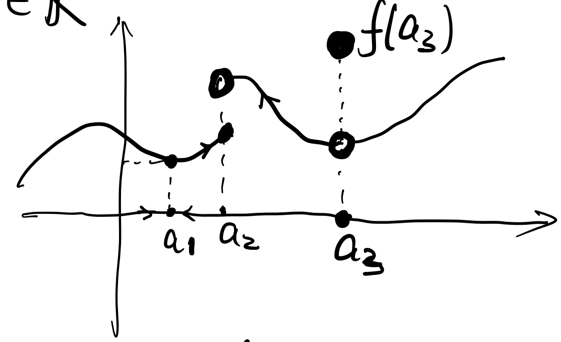
Multiply $2\cdot(x-2)^2$: $(-x-10)\cdot(x-2)>0$

$(x+10)(x-2)<0$



Limit: $f(x)$ $\quad a \in \mathbb{R}$

$$\lim_{x \to a} f(x)$$



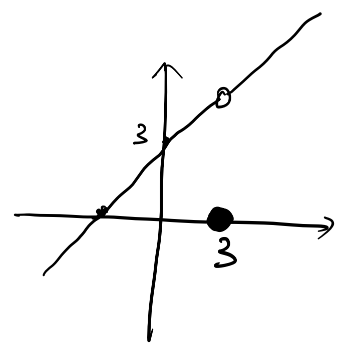
$$\boxed{\begin{aligned} &\lim_{x \to a_2^-} f(x) = f(a_2) \\ &\lim_{x \to a_2^+} f(x) \neq f(a_2) \\ &\lim_{x \to a_2} f(x) \text{ does not exist} \end{aligned}}$$

$$\lim_{x \to a_1} f(x) = f(a_1)$$

$$\lim_{x \to a_3} f(x) \neq f(a_3)$$

exist

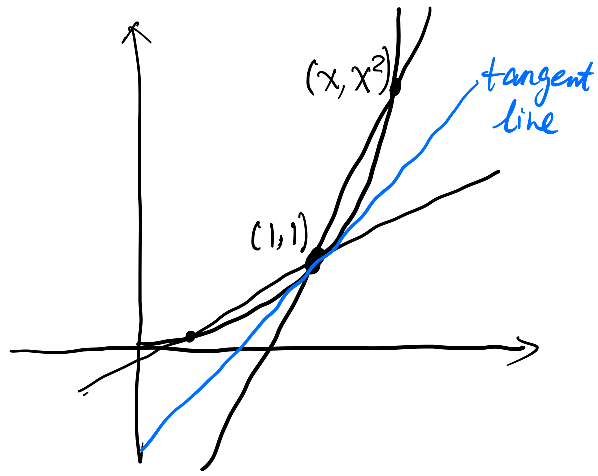
Ex: $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} = x+3, & (x \neq 3) \\ 0 & x = 3 \end{cases}$



$$\lim_{x \to 3} f(x) = \lim_{\substack{x \to 3 \\ (x \neq 3)}} (x+3) = 3+3 = 6 \neq f(3) = 0$$

Ex:

slope of line connecting $(1,1)$ and (x,x^2) $(x\neq 1)$



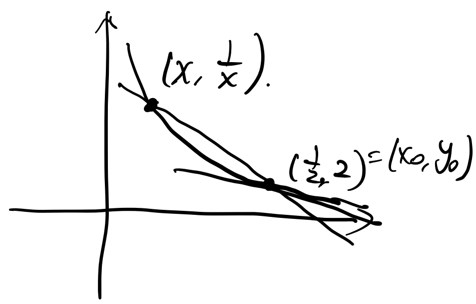
$$\boxed{k(x)=\frac{x^2-1}{x-1}=x+1}$$

$$\lim_{x\to 1} k(x)=\lim_{x\to 1}(x+1)=1+1=\underline{2}$$

tangent line: $y-1=2\cdot(x-1) \Leftrightarrow \boxed{y=2x-2+1=2x-1}$

Idea: limit of slopes of secant lines = slope of tangent line.

Ex: $y=\frac{1}{x}$, tangent line at point $\left(\frac{1}{2}, 2\right)$



slopes of secant lines:

$$\frac{\frac{1}{x}-2}{x-\frac{1}{2}}=\frac{\frac{1-2x}{x}}{\frac{2x-1}{2}}$$

$$\parallel$$

$$\underline{k(x)=-\frac{2}{x}}, \quad \lim_{x\to\frac{1}{2}} k(x)=-\frac{2}{\frac{1}{2}}=-4$$

$$\Rightarrow \quad y=-4\cdot\left(x-\frac{1}{2}\right)+2=-4x+4$$

$$=k\cdot(x-x_0)+y_0$$

$y = f(x)$

$$\boxed{k(h) = \frac{f(x+h) - f(x)}{h}}$$

$\lim_{h \to 0} k(h) =$ slope of tangent line