Math 311 Section 05

Quiz 8

Fall 2018

Name: ______________________________________

1. (1 point) __**T**__ True or False: If f is differentiable at x_0, then it is continuous there.
2. (1 point) __**F**__ True or False: If f is continuous at x_0, then it is differentiable there.
3. (1 point) __**F**__ True or False: If $f + g$ is differentiable, then so are f and g.
4. (1 point) Write the limit definition of the derivative of a function:

   $f'(x_0) =$

   Solution: $f'(x_0) = \lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

5. (1 point) Determine the error in the following argument. Remember that you are not being asked whether the statement is true. You are being asked to find the flaw in the logic of the proof.

   Claim 1. $\frac{1}{2}$ *is the supremum of* $\mathbb{Q}$.

   Proof. We will show this by the ε-characterization of supremums. Let $\varepsilon > 0$. Then, since the rational numbers are dense in $\mathbb{R}$, there exists some rational number $q \in \mathbb{Q}$ such that

   $$\frac{1}{2} - \varepsilon < q < \frac{1}{2}.$$

   Hence, $\frac{1}{2}$ is the supremum of $\mathbb{Q}$. □

   Solution: This ε-characterization only tells us a number is the supremum if we first know that it is an upper bound. Clearly, $\frac{1}{2}$ is not an upper bound for $\mathbb{Q}$, so it cannnot possibly be the supremum.