

Name: _____

1. (1 point) **F** True or False: For every $S \subseteq \mathbb{R}$, either S is closed or S is open.
2. (1 point) **T** True or False: The complement of a closed set is always open.
3. (1 point) **F** True or False: An open set doesn't contain any of its accumulation points.
4. (1 point) Define the set $S \subseteq \mathbb{R}$ as follows:

$$S = \bigcap_{x>0} (-x, x). \quad (1)$$

Is S open or closed? Give some reasoning for your claim, but it does *not* need to be a precise proof.

Solution: $S = \{0\}$, which is a singleton set and therefore is closed.

5. (1 point) Determine the error in the following argument:

Claim 1. For all $a, b \in \mathbb{N}$, if $a^2 | b^2$ then $a | b$.

Proof. Let $a, b \in \mathbb{N}$ with $a^2 | b^2$. Then, there exist an integer k such that

$$b^2 = ka^2.$$

Taking the square root of both sides, we get

$$b = \sqrt{k}a,$$

which proves that $a | b$. □

Solution: The definition of $a | b$ requires $\sqrt{k}$ to be an integer, which the author has not proven to be the case.

6. (1 point (bonus)) Is the empty set open or closed? You do *not* need to prove your answer.

Solution: The empty set is both open and closed. This is also true for $\mathbb{R}$. In fact, these are the only two sets which are both open and closed.