Math 311 Section 05 **Quiz 6** Fall 2018

Name: ______________________________

1. (1 point) __**F**__ Let $f, g : \mathbb{R} \to \mathbb{R}$ and define $h(x) = f(x)g(x)$. True or False: if h is continuous, then so are f and g.

2. (3 points) Using the ε, δ definition, prove that $f(x) = 2x + 3$ is continuous at $x_0 = 7$. *You must write the final proof, not just scratch work.*

Solution:

Proof. Let $\varepsilon > 0$. Let $\delta = \varepsilon/2$ and let $x \in \mathbb{R}$ be such that $|x - 7| < \delta$. Then

$$\left|f(x) - f(7)\right| = |2x + 3 - 17| = 2|x - 7| < 2\delta = \varepsilon.$$

□

3. (1 point) Determine the error in the following argument:

Claim 1. *The function $f(x) = x + 2$ is continuous at $x_0 = 0$.*

Proof. Let $\varepsilon = \frac{1}{2}$ and set $\delta = \frac{1}{2}$. Then, if $|x - x_0| = |x| < \delta = \frac{1}{2}$, then

$$\left|f(x) - f(0)\right| = \left|(x + 2) - (0 + 2)\right| = |x| < \frac{1}{2} = \varepsilon.$$

Hence, f is continuous at $x_0 = 0$. □

Solution: The definition of continuity has ε universally quantified, so you must prove the statement for all possible positive values of ε, not just $1/2$.