

Name: \_\_\_\_\_

1. (1 point) Which of the following kinds of statements can be *disproven* using a counterexample? Mark all that apply.

☒ **Universal statements** (e.g. “For all even numbers  $n$ ,  $n$  is divisible by 2.”)

☐ Existential statements (e.g. “There exists an even number divisible by 2.”)

☒ **Conditional statements** (e.g. “If  $n$  is even, then  $n$  is divisible by 2.”)

2. (1 point) F True or False: Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ . If  $\{x_n\}$  is some sequence in  $\mathbb{R}$  converging to  $x_0$  such that the sequence  $\{f(x_n)\}$  converges to  $L$ , then

$$\lim_{x \rightarrow x_0} f(x) = L.$$

3. (2 points) For what  $a \in \mathbb{R}$  does the limit  $\lim_{x \rightarrow a} \lfloor x \rfloor$  exist where  $\lfloor x \rfloor$  is the greatest integer less than or equal to  $x$ ? You do not need to provide a formal proof, but you must give some justification for your answer. (1 point for answer, 1 point for justification)

**Solution:** The limit exists if and only if  $a \notin \mathbb{Z}$ . At those points, the function jumps. (Drawing a graph is particularly helpful for this explanation)

4. (1 point) Determine the error in the following argument:

**Claim 1.** Let  $\{a_n\}$  and  $\{b_n\}$  be sequences of real numbers such that

$$\lim_{n \rightarrow \infty} (a_n - b_n) = 0.$$

Then  $a_n$  and  $b_n$  converge to the same number.

*Proof.*

$$0 = \lim_{n \rightarrow \infty} (a_n - b_n) = \lim_{n \rightarrow \infty} (a_n) - \lim_{n \rightarrow \infty} (b_n).$$

Hence, by moving the last term to the left hand side, we have

$$\lim_{n \rightarrow \infty} (a_n) = \lim_{n \rightarrow \infty} (b_n)$$

□

**Solution:** We know that the limit of the sum (or difference) is the sum (or difference) of the limits *as long as all of the limits exist*. There is no assumption here that the limits need exist, so we cannot split the limit as was done in the first line of the proof.