Name: ________________________________

1. (1 point) __**F**__ True or False: If $\lim_{x\to x_0} f(x) = L$ and x_0 is in the domain of f, then $f(x_0) = L$.

2. (1 point) Negate the following statement: For all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $|x - x_0| < \delta$ implies $\left|f(x) - L\right| < \varepsilon$.

   ◯ For all $\varepsilon \leq 0$, there exists $\delta \leq 0$ such that for all $x \in \mathbb{R}$, $|x - x_0| < \delta$ implies $\left|f(x) - L\right| < \varepsilon$.

   √ **There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| < \delta$ but $\left|f(x) - L\right| \geq \varepsilon$.**

   ◯ There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| > \delta$ and $\left|f(x) - L\right| > \varepsilon$.

   ◯ There exists $\varepsilon \leq 0$ such that for all $\delta \leq 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| < \delta$ but $\left|f(x) - L\right| \geq \varepsilon$.

3. (1 point) Define $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}.$$

   What is $\lim\limits_{x\to 0} f(x)$? You do *not* need to prove your answer.

   Solution: $\lim_{x\to 0} f(x) = 0$

4. (1 point) Describe the logical difference between the following two statements. (Note: simply stating that the quantifiers are rearranged is not sufficient. You should describe how that changes the logic of the statement.)

   Statement 1: For all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in \mathbb{R}$, $|x - y| < \delta$ implies $\left|f(x) - f(y)\right| < \varepsilon$.

   Statement 2: For all $\varepsilon > 0$ and for all $y \in \mathbb{R}$, there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $|x - y| < \delta$ implies $\left|f(x) - f(y)\right| < \varepsilon$.

   Solution: In Statement 1, the same δ must work for all y. In Statement 2, it is possible that we require different δs for different values of y.

5. (1 point) Determine the error in the following argument:

   Claim 1. *Let $n \in \mathbb{Z}$. If $n^2 + 2n + 4$ is divisible by 4, then n is even.*

   Proof. Suppose $n \in \mathbb{Z}$ is even. Then there is an integer k such that $n = 2k$. So,

$$n^2 + 2n + 4 = 4k^2 + 4k + 4 = 4(k^2 + k + 1).$$

   Since $k^2 + k + 1 \in \mathbb{Z}$, this shows that $n^2 + 2n + 4$ is divisible by 4. □

Solution: This is a proof of the converse, which is *not* logically equivalent to the original statement.