Practice Test I, Math 292 Spring 2014

Eric A. Carlen[1]
Rutgers University

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1: **(a)** Find the general solution of

$$x'(t) = -x(t) + \sin t \tag{0.1}$$

and find the unique value of $x(0)$ for which the solution is periodic.

(b) Let $x_1(t)$ and $x_2(t)$ be any two solutions of (0.1). Show that for all $\lambda < 1$,

$$\lim_{t\to\infty} e^{t\lambda}|x_1(t) - x_2(t)| = 0 \ .$$

2: Consider the two differential equations

(i) $x'(t) = \sqrt{1 - x^2(t)}$, $\qquad x(0) = 0$

and

(ii) $y'(t) = 1 - y^2(y)$, $\qquad y(0) = 0$

One of the two solutions reaches zero in a finite time T. That is there is a T such that $x(T) = 1$ or there is a T such that $y(T) = 1$, Which one is it, and what is T?

3: Consider the Ricatti equation

$$x'(t) + x^2(t) = \frac{2}{t^2} \tag{0.2}$$

for $t > 0$.

(a) Find a particular solution $x_1(t)$ of the form $x_1(t) = Ct^\alpha$.

(b) Find the general solution.

(c) Find the solution that satisfies $x(1) = 1$ and the interval on which this soluiton is defined.

4: Consider the vector field

$$\mathbf{v}(x, y) = ((y - x)(1 - x - y) \ , \ x(2 + y)) \ .$$

(a) Find all equlibrium points, and for each one, determine whether it is assymptoically stable, Lyapunov stabe or unstable. Explain your reasining and justify your answer with appropriate calculuations.

(b) Sketch the solution curves in the vicinity of each equilibrium point.

5: Consider the system

$$\begin{aligned} x' &= -4x - 9y \\ y' &= x + 2y \end{aligned} \tag{0.3}$$

(a) Find a matrix A so that this system can be written as $\mathbf{x}' = A\mathbf{x}$, and compute e^{tA}.

(b) Let $bx(t) = (x(t), y(t))$ be the solution with $(x(0), y(0)) = (x_0, y_0)$. Find all values of x_0 and y_0 such that

$$\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}\ .$$

(c) Use Duhamel's formula to find the solution of

$$\mathbf{x}'(t) = A\mathbf{x}(t) + (0, t)$$

with $\mathbf{x}(0) = (1, 1)$.