Homework Assignment 7, Math 292, Spring 2014

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1. Let

$$I[y] = \int_1^2 \frac{\sqrt{1+y^2}}{y} \mathrm{d}x$$

Consider the problem of minimizing $I[y]$ subject to $y(1) = a$ and $y(2) = b$. Find the corresponding solution, or solutions, of the Euler-Lagrange equation.

2. Let

$$I[y] = \int_0^4 [xy' - (y')^2] \mathrm{d}x$$

Consider the problem of maximizing $I[y]$ subject to $y(0) = 0$ and $y(4) = 3$. Find the corresponding solution, or solutions, of the Euler-Lagrange equation. Also, explain why no minimum exists.

3. Consider the problem of finding a curve $y(x)$ with $y(0) = 1, y(1) = 0$, and such that

$$\int_0^1 \sqrt{1 + (y')^2} \mathrm{d}x = L$$

with L given, and such that the area under the curve and above the x-axis is minimal. For which values of L does such a curve exist, and what is it?

5. Find the Green's function for

$$\mathcal{L}u = ((1+x)^2 u')' - u$$

subject to $u(0) = u(1) = 0$, and solve

$$((1+x)^2 u')' - u = e^x$$

5. Let $L > 0$ and let

$$I[y] = \int_0^L [(y')^2 - y^2 - (\sin x) y] \mathrm{d}x \ .$$

Consider the problem of minimizing $I[y]$ subject to $y(0) = y(L) = 0$. Find the corresponding Euler-Lagrange equation. For which values of L does it have a solution subject to the boundary conditions? What is the greatest lower bound as a function of L, and for which values of L is it a minimum?