Homework Assignment 5, Math 292, Spring 2014

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1. Find the exact solution of $x'(t) = v(x(t), t)$ and $x(0) = 0$ for

$$v(x,t) = 2t(1+x) \ .$$

Stating from $X_0 = 0$, compute the next 4 terms in the Picard iteration, namely X_1, X_2, X_3 and X_4, and compare with the exact solution.

2. Consider the non-autonomous first order system $\mathbf{x}'(t) = A(t)\mathbf{x}(t)$ with

$$A(t) = \begin{bmatrix} 0 & 1 \\ t^{-2} & -t^{-1} \end{bmatrix} \ .$$

We have found that the flow transforation has the matrix $[\Phi_{t,s}]$ where

$$[\Phi_{t,s}] = M(t)M^{-1}(s) = \frac{1}{2}\begin{bmatrix} \frac{t^2+s^2}{st} & \frac{t^2-s^2}{t} \\ \frac{t^2-s^2}{st^2} & \frac{t^2+s^2}{t^2} \end{bmatrix} \ .$$

Starting from $\mathbf{X}_0 = \mathbf{x}_0$ for general $\mathbf{x}_0 \in \mathbb{R}^2$, compute the next 3 terms in the Picard iteration, namely $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$ for the solution of $\mathbf{x}'(t) = A(t)\mathbf{x}(t)$ and $\mathbf{x}(s) = \mathbf{x}_0$. Compare the first three Picard iterates for the flow with the exact flow.

3. Find the general solution of

$$t^2x''(t) + tx'(t) - 4x(t) = t \ .$$

Also, compute the flow transformation of the first order linear system corresponding to

$$t^2x''(t) + tx'(t) - 4x(t) = 0 \ .$$

4. Find the general solution of

$$x''(t) - x'(t) - 6x(t) = e^t \ .$$

5. Find the general solution of

$$(t^2+t)x'' + (2-t^2)x' - (2+t)x = t(t+1)^2 \ .$$