Practice Test 1B, Math 292 Spring 2013

Eric A. Carlen[1]
Rutgers University

March 10, 2013

(1) Find the general solution of the equation

$$x'(t) = x^2(t) + tx(t) - (1+t)\ ,$$

and the particular solution with $x(0) = 1$.

(2.) Let $v(x) = 4x - x^2$.

(a) Show that for each $x_0 \in (0,4)$ there exist a unique solution of

$$x'(t) = v(x) \qquad \text{with} \quad x(t) = x_0\ . \tag{$*$}$$

(b) How long does it take the solution of $(*)$ with $x_0 = 1$ to reach $x = 3$?

(c) Find an explicit formula for the flow transformation φ_f such that for each $x_0 \in (0,4)$, $x(t) = \varphi_t(x_0)$ is the solution to $(*)$. Compute $\varphi_t'(1)$.

(3.) Let

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix} \qquad \text{and} \qquad B := \begin{bmatrix} 0 & -2 \\ 1 & -2 \end{bmatrix}\ .$$

(a) Compute e^{tA} and e^{tB}.

(b) Find all $\mathbf{x}_0$, if any, so the the solution of $\mathbf{x}'(t) = A\mathbf{x}(t)$ with $\mathbf{x}(0) = \mathbf{x}_0$ satisfies $\lim_{t\to\infty} \mathbf{x}(t) = 0$.

(c) Find all $\mathbf{x}_0$, if any, so the the solution of $\mathbf{x}'(t) = B\mathbf{x}(t)$ with $\mathbf{x}(0) = \mathbf{x}_0$ satisfies $\lim_{t\to\infty} \mathbf{x}(t) = 0$.

(d) Let $\mathbf{f}(t) = (1,t)$, and let $\mathbf{x}_0 = (2,1)$. Find the solution of

$$\mathbf{x}'(t) = B\mathbf{x}(t) + \mathbf{f}(t)$$

with $\mathbf{x}(0) = \mathbf{x}_0$.

Find all $\mathbf{x}_0$, if any, so the the solution of $\mathbf{x}'(t) = B\mathbf{x}(t)$ with $\mathbf{x}(0) = \mathbf{x}_0$ satisfies $\lim_{t\to\infty} \mathbf{x}(t) = 0$.

(4.) Consider the *driving force* $\mathbf{f} = 3\cos(\omega t)(1,2)$, where $\omega > 0$. Find the solution of

$$M\mathbf{x}'' = -A\mathbf{x} + \mathbf{f} \qquad \text{with} \qquad \mathbf{x}(0) = \mathbf{0}\ , \quad \mathbf{x}'(0) = \mathbf{0}\ . \tag{0.1}$$

where

$$M = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 12 & 2 \\ 2 & 3 \end{bmatrix}\ .$$