Practice Test for Test 2, Math 292, April 25, 2013

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1. The differential equation

$$t^2 x''(t) - 3tx'(t) + 4x(t) = 0$$

has polynomial coefficients.

(a) Find one polynomial solution to this equation.

(b) Find the general solution of this equation.

(b) Find the general solution of

$$t^2 x''(t) - 3tx'(t) + 4x(t) = t^2 \ln t \ .$$

2. Consider the differential equation

$$y'(x) = f(x,y) \qquad \text{where} \qquad f(x,y) = -\frac{1}{x^3} - \frac{2}{x}y + xy^2 \ . \tag{0.1}$$

Consider also the change of variables

$$h(x,y) = (u(x,y), v(x,y)) = (-\ln x, x^2 y). \tag{0.2}$$

(a) Compute the transformed slope field $h_*(1,f)(u,v)$, and find the general solution of the transformed equation.

(b) Find the general solution of the equation (0.1).

3. Consider the equation

$$y''(x) - xy'(x) + \frac{x^2}{2}y(x) = 0 \ . \tag{0.3}$$

(a) Find a function $q(x)$ so that whenever $y(x)$ is a solution of (0.2), there is a solution $z(x)$ of

$$z''(x) + q(x)z(x) = 0 \tag{0.4}$$

that has the same set of zeros as $y(x)$.

(b) Find a number $L > 0$ so that if $y(x)$ solves (0.3) and satisfies $y(0) = 0$ and $y'(0) = 1$, then for some x_1 with $0 < x_1 < L$, $y(x_1) = 0$. Justify your answer.

4. Find the continuously differentiable curve $y(x)$ such that $y(0) = 1$ and $y(1) = 0$ that minimizes the functional

$$I[y] = \int_0^1 [|y'(x)|^2 + |y(x)|^2]\mathrm{d}x \ .$$

Justify your answer.