Homework Set 3, Math 292 Spring 2013

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February 17, 2013

These exercises are due Thursday, February 28.

1. Consider the system of equations

$$\mathbf{x}' = A\mathbf{x}$$

where

$$A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix} .$$

(a) Find the flow transformation Φ_t for this system, which, since this is a linear system is $\Phi_t(\mathbf{x}) = e^{tA}\mathbf{x}$.

(b) Note that $\mathbf{0}$ is an equilibrium point for the vector field $\mathbf{v}(\mathbf{x}) = A\mathbf{x}$. Is it Lyapunov stable, asymptotically stable or unstable? (To decide this, look at how $e^{ta}\mathbf{x}$ behaves for large t.)

2. Consider the system of equations

$$\mathbf{x}' = A\mathbf{x}$$

where

$$A = \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix} .$$

(a) Find the flow transformation Φ_t for this system, which, since this is a linear system is $\Phi_t(\mathbf{x}) = e^{tA}\mathbf{x}$.

(b) Note that $\mathbf{0}$ is an equilibrium point for the vector field $\mathbf{v}(\mathbf{x}) = A\mathbf{x}$. Is it Lyapunov stable, asymptotically stable or unstable? (To decide this, look at how $e^{ta}\mathbf{x}$ behaves for large t.)

3. Consider the system of equations

$$\mathbf{x}' = A\mathbf{x}$$

where

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 4 \\ 0 & 0 & -2 \end{bmatrix} .$$

(a) Find the flow transformation Φ_t for this system, which, since this is a linear system is $\Phi_t(\mathbf{x}) = e^{tA}\mathbf{x}$.

(b) Note that $\mathbf{0}$ is an equilibrium point for the vector field $\mathbf{v}(\mathbf{x}) = A\mathbf{x}$. Is it Lyapunov stable, asymptotically stable or unstable? (To decide this, look at how $e^{ta}\mathbf{x}$ behaves for large t.)

4. Consider the system of equations

$$\mathbf{x}' = A\mathbf{x}$$

where

$$A = \begin{bmatrix} -5 & 3 & 2 \\ -8 & 5 & 4 \\ 4 & -3 & -3 \end{bmatrix} .$$

(a) Find the flow transformation Φ_t for this system, which, since this is a linear system is $\Phi_t(\mathbf{x}) = e^{tA}\mathbf{x}$.

(b) Find all values of $\mathbf{x}_0$ such that the solution of $\mathbf{x}'(t) = A\mathbf{x}(t)$ with $\mathbf{x}(0) = \mathbf{x}_0$ satisfies

$$\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0} .$$