Practice Test IB, Math 291 Fall2013

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1: Let $\mathbf{a} = (1, 1, 1)$

(a) Find a vector $\mathbf{x}$ such that

$$\mathbf{x} \times \mathbf{a} = (-7, 2, 5) \qquad \text{and} \qquad \mathbf{x} \cdot \mathbf{a} = 0 \ .$$

(b) There is no vector $\mathbf{x}$ such that

$$\mathbf{x} \times \mathbf{a} = (1, 0, 0) \qquad \text{and} \qquad \mathbf{x} \cdot \mathbf{a} = 0 \ .$$

Show that no such vector exists.

2: Let P_1 denote the plane through the three points $\mathbf{a_1} = (1, 2, 1)$ $\mathbf{a_2} = (-1, 2, -3)$ and $\mathbf{a_3} = (2, -3, -2)$. Let P_2 denote the plane through the three points $\mathbf{b_1} = (1, 1, 0)$ $\mathbf{b_2} = (1, 0, 1)$ and $\mathbf{b_3} = (0, 1, 1)$.

(a) Find equations for the planes P_1 and P_2.

(b) Parameterize the line given by $P_1 \cap P_2$, and find the distance between this line and the point $\mathbf{a_1}$.

(c) Consider the line through $\mathbf{b_1}$ and $\mathbf{b_2}$. Determine the point of intersection of this line with the plane P_1, and find a parametrization of the line given by reflecting this line off plane P_1. That is, the direction vector of the reflected line is what you get by reflecting the direction vector of the original line using the unit normal direction $\mathbf{U}$ to the plane.

3: Let $\mathbf{x}(t)$ be the curve given by

$$\mathbf{x}(t) = (e^t \cos(t), e^t \sin(t), e^t) \ .$$

(a) Compute the arc length $s(t)$ as a function of t, measured from the starting point $\mathbf{x}(0)$, and find an arc-length parameterization of this curve

(b) Compute curvature $\kappa(t)$ and torsion $\tau(t)$ as a function of t.

(c) Find an equation for the osculating plane at time $t = 0$

4: **(a)** Let $f(x, y)$ be given by

$$f(x,y) = \begin{cases} \dfrac{xy}{|x| + |y|} & (x,y) \neq (0,0) \\ 0 & (x,y) \neq (0,0) \ . \end{cases}$$

Does

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

exist? If so, evaluate the limit. If not, explain why not.

(b) Let $g(x,y)$ be given by

$$g(x,y) = \begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^4+1}-1} & (x,y) \neq (0,0) \\ 0 & (x,y) \neq (0,0) \ . \end{cases}$$

Does

$$\lim_{(x,y)\to(0,0)} g(x,y)$$

exist? If so, evaluate the limit. If not, explain why not.

5: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = x^2y + yx - xy^2 \ .$$

(a) Compute the gradient of f, and find all critical points of f..

(b) Find the equation of the tangent plane to the graph f at the point $(1,1)$.

(c) Let $\mathbf{x}(t) = (1+t-t^3, 2-t+t^2)$. Compute $\left.\dfrac{\mathrm{d}}{\mathrm{d}t} f(\mathbf{x}(t))\right|_{t=0}$.

(d) Find all points (x,y) at which the tangent plane to the graph of f is orthogonal to the line parameterized by $t(1,0,1)$.

Extra Credit: Let $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$ be three vectors in $\mathbb{R}^3$ such that $\mathbf{w}_2 \times \mathbf{w}_3 \neq \mathbf{0}$. Consider the curve

$$\mathbf{x}(t) = t\mathbf{w}_1 + t^2\mathbf{w}_2 + t^3\mathbf{w}_3 \ .$$

Show that this curve is planar if $\mathbf{w}_1 \cdot \mathbf{w}_2 \times \mathbf{w}_3 = 0$, and for more extra credit, show that "if" can be upgraded to "if and only if".