Practice Test IIB, Math 291 Fall 2010

December 1, 2013

1: Let $f(x,y) = x^2y + xy^2 - xy$.

(a) Find all of the critical points of f. Evaluate the Hessian matrix of f at each of these critical points, and for each, determine whether it is a local maximum, a local minimum, a saddle, or if this is undecidable from the Hessian.

(b) There is one critical point in the interior of the upper right quadrant. Sketch a contour plot of f in the vicinity of this critical point. Show the computations that lead to the plot to get credit.

(c) Find all unit vectors $\mathbf{u}$ that make

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} f((1/3, 1/3) + t\mathbf{u})\bigg|_{t=0}$$

as *large* as possbile.

2: Let $f(x,y) = xy + 2x - 2y$. *Use the method of Lagrange* to find the minimum and maximum values of f on the region $\Omega \subset \mathbb{R}^2$ that lies below the parabola $y = 3 - x^2$ and above the line $y = 2x$.*(You must use the method of Lagrange, showing your steps, for credit. This problem is designed to work out nicely, and can be done by other means, but ony solutions based on Lagrange's method will earn credit.)*

3: Let $f(x,y)$ and $g(x,y)$ be given by

$$f(x,y) = y - x^5 \qquad \text{and} \qquad g(x,y) = x^2 + y^2 - 5 \ .$$

Define the function $\mathbf{f}$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ by

$$\mathbf{f}(\mathbf{x}) = (f(\mathbf{x}), g(\mathbf{x})) \ .$$

(a) Sketch the curves given by $f = 0$ and $g = 0$. How many solutions are there to the system $\mathbf{f} = \mathbf{0}$?

(b) Compute the Jacobian of $\mathbf{f}$; i.e., $[D_{\mathbf{f}}(\mathbf{x})]$.

(c) Use $\mathbf{x}_0 = (1,2)$ as a starting point for Newton's method for solving $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, and find $\mathbf{x}_1$, the next step. Evaluate $\mathbf{f}(\mathbf{x}_1)$. Comment on your result.

4: **(a)** Let $D \subset \mathbb{R}^2$ be the region that is to the right of the parabola $x = (y-1)^2$ and below the line $x + 2y = 5$. Compute $\int_D xy\mathrm{d}A$.

(b) Let D be the region in $\mathbb{R}^2$ that consisting of points (x,y) such that

$$1 \le xy \le 2 \quad \text{and} \qquad x^2 \le y \le 2x^2 \ .$$

Compute $\int_D xy \mathrm{d}A$.

5: Let $\mathcal{V}$ be the region in $\mathbb{R}^3$ that is inside the ellipsoid $4x^2 + 4y^2 + z^2 = 5$, and above the plane $z = 1$. Let $\mathcal{S}$ be the boundary of $\mathcal{V}$.

(a) Compute the integral

$$\int_{\mathcal{V}} z \mathrm{d}V \ .$$

(b) Compute the surface area of $\mathcal{S}$. You may express your answer as a defnit integral of a function of one real variuable. (The final integral can be done in explicitly if you remember your standars substitutions, but this is not the point of the question.)