Practice Test IIA, Math 291 Fall 2013

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1: Let $f(x,y) = x^2 + y^2 - 2yx^2$.

(a) Find all of the critical points of f. Evaluate the Hessian matrix of f at each of these critical points, and determine where each is a local maximum, a local minimum, a saddle, or undecidable from the Hessian.

(b) Sketch a contour plot of f in the vicinity of each of the critical points. Show the computations that lead to the plots to get credit.

2: Let $f(x,y) = xy$. Let D denote the region in the plane consisting of all of the points (x,y) such that

$$x^2 + 4y^2 \leq 6 \ .$$

Find the minimum and maximum values of f in D. Also, find all of the minimizers and maximizers in D.

3: Let $\mathbf{f}(\mathbf{x}) = (f(\mathbf{x}), g(\mathbf{x}))$ where $f(x,y) = xy - x^3 - 1/4$, and $g(x,y) = 1 - 4y^2 - x^2$.

(a) How many solutions to the system $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ are there? Draw a plot showing their approximate location.

(b) In the previous part, you should have found that there is one solution not too far from

$$\mathbf{x}_0 = (-1, 1/2) \ .$$

Compute $[D_{\mathbf{f}}(\mathbf{x})]$, and then use $\mathbf{x}_0$ as a starting point for Newton's method, and compute the next approximate solution $\mathbf{x}_1$.

(c) Evaluate $\mathbf{f}(\mathbf{x}_1)$, and compare this with $\mathbf{f}(\mathbf{x}_0)$.

4: Let D be the set in $\mathbb{R}^2$ given by

$$2x^2 + 2xy + 2y^2 \leq 1 \ .$$

Let $f(x,y) = x^2 + y^2$. Compute $\int_D f(x,y)\mathrm{d}A$. (**Hint**: Find a change of coordinate $(u(x,y), v(x,y)$ under which $2x^2 + 2xy + 2y^2 = 1$ becomes $u^2 + v^2 = 1$.)

5: (a) Let $\mathcal{V}$ be the region in $\mathbb{R}^3$ that lies below the graph of $z = 1 - x^2$, and above the graph of $z = y^2$. Compute the volume of $\mathcal{V}$.

(b) Let $\mathcal{S}$ be the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the plane $x + z = 1$. Compute $\int_{\mathcal{S}} f(x, y, z)\mathrm{d}S$ where $f(x, y, z) = y/\sqrt{x^2 + y^2}$. To get full credit, carry the computations through to the point that only an integral over a single variable remains to be evaluated.

Extra Credit: $\mathcal{S}$ be upper hemisphere of the unit sphere in $\mathbb{R}^3$. Let $f(x, y, z) = xyz$. Find the minimum and maximum values of f on $\mathcal{S}$, and all of the points at which f takes on these values. Explain how you are taking into account both of the constraints $x^2 + y^2 + z^2 = 1$ and $z \geq 0$.