Question 1

Let p be a prime and let $\mathbb{Z}_{(p)}$ be the subset of the rational numbers consisting of the numbers a/b such that p does not divide b. These are the *p-local integers*.

(a) Show that $\mathbb{Z}_{(p)}$ is a subring of $\mathbb{Q}$.

(b) Let $a/b \in \mathbb{Z}_{(p)}$. Show that a/b is a unit iff p does not divide a.

(c) Show that every element $r \in \mathbb{Z}_{(p)}$ determines an unique unit $u \in \mathbb{Z}_{(p)}$ and a unique natural number n such that $r = up^n$.

(d) Show that every element $q \in \mathbb{Q}$ determines an unique unit $u \in \mathbb{Z}_{(p)}$ and a unique integer n such that $q = up^n$.

(e) Find a surjective ring homomorphism $\pi : \mathbb{Z}_{(p)} \to \mathbb{Z}_p$. Make sure to show that it is well-defined.