Question 1

Let F be a field (for now, one of $\mathbb{Q}, \mathbb{R}, \mathbb{C}$, or $\mathbb{Z}_p$ for a prime p).

Let $\mathbb{A} = AGL(1, F)$ be the *one-dimensional affine group* over F: the set of maps $f : F \to F$ of the form $f(x) = ax + b$, with $a, b \in F$ and a a unit.

Let S be the subset of *scalings*: maps of the form $s(x) = ax$.

Let T be the subset of *translations*: maps of the form $t(x) = x + b$.

(a) Show that $\mathbb{A}$ is a group and that S and T are subgroups.

(b) Is S normal? Is T normal?

(c) When $F = \mathbb{Z}_p$, find the order of $\mathbb{A}$.
Let D be the subgroup generated by T together with the single scaling $s(x) = -x$.
Which familiar group is D isomorphic to?