Question 1

An *automorphism* of a group G is an isomorphism from G to itself. Denote the set of automorphisms of G by $Aut(G)$.

(a) Given $x \in G$, define a map $c_x : G \to G$ by $c_x(g) = x^{-1}gx$. Prove that this is an automorphism of G. (We call these *inner* isomorphisms, and denote the set of such by $Inn(G)$).

(b) Prove that $Aut(G)$ is a group and $Inn(G)$ is a subgroup.

(c) Describe $Inn(G)$ when G is abelian.

(d) Consider the map $G \to Inn(G)$, $x \to c_x$. Prove that this is a surjective homomorphism with kernel $Z(G)$ (the *center* of G).