Question 1

Let G be a group. Recall that for $a \in G$, we write $|a|$ for the order of a (the least positive integer n such that $a^n = e$, or ∞ if no such n exists).

(a) Given $a \in G$, prove that $|a| = |a^{-1}|$. (Be careful about the case where a is of infinite order).

(b) Given commuting elements $a, b \in G$, both of finite order, prove that $|ab|$ divides $lcm(|a|, |b|)$.

(c) Let D be the group of symmetries of the real line that take integers to integers. Find distinct elements $a, b \in D$, both of order 2. What is the order of ab? (D is the *infinite dihedral group*. Think of the the real line as a regular n-gon, with $n = \infty$).