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*This issue (Number 1) and the next issue (Number 2) of Chinese Annals of Mathematics Series B are dedicated to Professor Haim Brezis on the occasion of his 70th birthday*

Avec notre admiration, notre reconnaissance et notre amitié,

Henri Jean-Michel

The Guest Editors

Henri BERESTYCKI Jean-Michel CORON

# Preface

Henri BERESTYCKI Jean-Michel CORON

It is with pleasure that we accepted the invitation of Tatsien Li, the Editor in chief of Chinese Annals of Mathematics (CAM), to act as guest editors of this journal for two special issues dedicated to Haim Brezis. The present issue and the next one are the result of this endeavor. We are delighted to gather in it contributions on current and future trends in PDEs and nonlinear analysis as a tribute to Haim's outstanding influence on theses fields.

With his legendary energy, Haim has written or co-authored over 250 papers and he is the author of three books. His inspiring enthusiasm and elegant lectures have attracted numerous students and others to his fields of interest and made him an international ambassador for nonlinear analysis and PDEs. Among his 57 Ph.D. students, many went on to become very active and well known researchers of their own. He has further contributed to mathematics in important editorial positions on journals and book series as well as professional societies. All of this has made Haim Brezis one of the most influential mathematicians of the last fifty years.

Haim Brezis has made fundamental contributions to a broad range of topics. It would be a daunting task to try to present an exhaustive list of Haim's mathematical achievements. In this preface we would like to mention some of them. Brezis began his mathematical career at a time when the abstract functional analysis approach to PDEs was at its height. In the late 1960's and early 1970's, the theory of monotone operators and nonlinear semi-groups was being developed under the influence of F. Browder, Crandall, Minty, Pazy and many others. Very early on, Brezis asserted himself as a top world expert of the subject, and his work instantaneously became a major reference on this topic. In a striking first article [19], he introduced two important classes of operators between spaces in duality: operators of type M and pseudo-monotone operators. Brezis then wrote a series of deep articles associated with regularizing effects of such semigroups when their generator is the sub-differential of a convex lower semicontinuous function in a Hilbert space. A further series of papers (in particular [28, 33]) treated the sum of monotone operators and gave sufficient conditions for their sum to be maximal monotone. These conditions are associated to the notion of dominant operator and they led to important progress in the abstract theory of operators. Together with Ivar Ekeland, Haim found a very surprising variational formulation for evolution equations involving the sub-differential of a convex function [29]. It is only relatively recently that people have started to realize the importance of this work, written as a short note to the Comptes-rendus of the Académir of sciences in Paris. Indeed, it comes up in many different frameworks.

Brezis' book [21] on monotone operators in which he presented the most important results, many of them his own, became an instant classic on the subject. The book already exemplifies three essential characteristic features of Haim's teaching and writing: (i) exploring a subject in depth, (ii) finding the most elegant and beautiful approach to it, and (iii) a desire to share with students and fellow researchers the beauty of the topic, that is, to welcome others to the state of the art knowledge on a subject. This book remains a central reference for researchers and graduate students interested in nonlinear operators.

His Doctoral thesis "Problèmes unilatéraux", published in 1972 in a 168 pages article [20], was concerned with the regularity theory of variational inequalities. In this work he showed how fractional Besov-Sobolev spaces or BV spaces are well adapted to study optimal regularity questions in nonlinear equations. Identifying the optimal regularity and characterizing the proper function space became a recurrent theme in his research. A series of visits to Pisa early on in his career where he met G. Stampacchia had a lasting influence on the work of Brezis dealing with regularity.

With several collaborators [37, 6], Brezis made important studies of semilinear elliptic equations with data in $L^1$ and more generally data that are merely measures, obtaining both existence results and giving criteria for when there are no solutions with isolated singularities. The latter type of result has an enormous number of extensions to a variety of problems such as quasilinear or fully nonlinear operators, parabolic operators, boundary singularities, higher order equations, systems, etc.

In a major piece of work, Haim Brezis with Louis Nirenberg [34] studied elliptic equations with power nonlinearity at the Sobolev cut-off point, resulting in a lack of compactness for the problem. They found a mild criterion for a solution to exist. An open case was solved later by Brezis [23]. The two articles [34] and [23] illuminated quantization effects in the study of functionals with critical Sobolev exponents as well as the importance of geometrical effects. These two papers alone have received more than 1000 citations on mathscinet, and have been the source of inspiration for a vast literature in an area that continues to be the subject of research to this day.

The theme of analyzing problems with a lack of compactness became a fixture of the work of Brezis for a decade after this. Many such problems arise in geometry. With J.-M. Coron, Brezis [26] solved a conjecture due to Rellich about surfaces of prescribed constant mean curvature: given a Jordan curve $\Gamma$ contained in the unit ball of $\mathbb{R}^3$, for all $0 < H < 1$ there exist two surfaces having constant mean curvature $H$ spanning $\Gamma$. Before their result the existence of only one such surface was known in general (Heinz, Hildebrandt). Brezis and Coron obtained this result through a very refined analysis of the lack of compactness in Palais-Smale sequences (to obtain the second surface as a saddle point). In the spirit of works of T. Aubin (Yamabe problem, best constant for the Sobolev embedding), of Brezis and Nirenberg (on the critical exponent mentioned above), and of Wente (previous works on Rellich), Brezis and Coron showed that this lack of compactness does not appear below a precise threshold. Then, by careful and delicate estimates, they proved that the candidate for a critical value of the functional obtained by applying the mountain pass strategy is below this threshold.

In [27] Brezis, J.-M. Coron, and E. Lieb introduced some new important ideas for harmonic mappings. They classified the possible singularities of minimizing harmonic mappings from $u : B^3 \mapsto \mathbb{S}^2$. Since the celebrated work of Schoen and Uhlenbeck it was known that such mappings have only finitely many singularities and that (say near $x = 0$) these singularities are locally of the type $\omega(\frac{x}{|x|})$ where $\omega(\frac{x}{|x|})$ is a minimizing harmonic map. Brezis, Coron and Lieb in [27] improved this result by showing that $\omega$ is a rotation, and that indeed the mapping $\frac{x}{|x|}$ is minimizing from $B^3 \to \mathbb{S}^2$. This work opened the way to much further study by others. Furthermore, in the same work, Brezis, Coron and Lieb introduced the notion of minimal connexion length between singular points which turned out to be very useful for the study of harmonic mappings and led to optimal inequalities involving the length and energy.

In collaboration which F. Bethuel and F. Hélein, Brezis initiated the study of the Ginzburg-Landau energy

$$\mathcal{E}_\varepsilon(u) := \frac{1}{2}\int_\Omega |\nabla u|^2 \,\mathrm{d}x + \frac{1}{4\varepsilon^2}\int_\Omega \left(1 - |u|^2\right)^2 \mathrm{d}x, \quad u : \Omega \subset \mathbb{R}^2 \to \mathbb{C},$$

which is related to phase transition phenomena in superconductors and superfluids. The book [8] they published on this topic has become a milestone for the subject. One of their main results asserted that that minimizers of $\mathcal{E}_\varepsilon$ enjoy some uniform (with respect to $\varepsilon$) Sobolev regularity, and from this fact they deduced that the limit of minimizers exists. Moreover, such a limit is smooth except at a finite number of points, called "defects" or "vortices" in physics, and the number of these defects is exactly the Brouwer degree of the boundary condition. Also, each singularity has degree one (or, as physicists say, vortices are quantized).

Note that, as $\varepsilon \to 0$, any limit $u$ of minimizers of $\mathcal{E}_\varepsilon$ satisfies the constraint $|u| = 1$ a.e. This fact motivated the study of Sobolev spaces with values in $\mathbb{S}^1$, and the development of a degree theory in this context. In two dimensions, natural Dirichlet data for the Ginzburg-Landau equation belongs to $H^{\frac{1}{2}}(\partial\Omega; \mathbb{S}^1)$, which is a subspace of the functions with vanishing mean oscillations VMO($\partial\Omega$). When the boundary condition has zero.degree, the limiting maps belongs to $H^1(\Omega; \mathbb{S}^1)$. This is again a subspace of VMO. Other geometric PDEs lead to VMO maps, e.g. the study of harmonic maps in two dimensions [25].

In two beautiful papers with Nirenberg, [35, 36], Brezis founded the theory of degree for VMO maps between manifolds. Based in part on previous works of Schoen and Uhlenbeck [48] and of Boutet de Monvel and Gabber [18], they established existence of the degree, as well as its main properties. This is the VMO counterpart of the degree theory for continuous maps. Their papers shed a new light on older results like the index theory for Toeplitz operators on the circle [41, Chapter 7] and on the BMO control of the lifting of $\mathbb{S}^1$-valued maps discovered by Coifman and Meyer [40]. Also, this extension of the notion of degree finally clarified a series of many partial results which had been previously proved in particular cases and in which no general argument could be used.

In [12], Bourgain, Brezis and Mironescu solved the lifting problem for $\mathbb{S}^1$-valued maps, obtaining sharp results about the possibility of writing a Sobolev map $u : \Omega \to \mathbb{S}^1$ in the form $u = e^{i\varphi}$ for some map $\varphi : \Omega \to \mathbb{R}$ which enjoys the same Sobolev regularity of $u$. This question is extremely delicate when the map has low regularity, and this paper paved the way to a considerable number of developments, e.g. lifting in covering spaces studied by Bethuel and Chiron [9], optimal estimates for liftings [11, 46], and applications to the De Gennes model in nematics [5].

The work of Bourgain, Brezis, and Mironescu [12] led Brezis in a new direction. With Li [30], he initiated the study of the topology of the space $W^{1,p}(M,N)$ itself. This work is also related to the work of B. White [49] on the existence of topological invariants for maps in the Sobolev space $W^{1,p}(M,N)$, with $N$ a compact oriented manifold. A striking result in the deep and novel paper of Brezis and Li is that when $\dim M \geq 2$ and $p < 2$, then $W^{1,p}(M,N)$ is path-connected. Their paper introduced useful new techniques for approximating and connecting maps. For some special $M$'s and $N$'s, they obtained two completely new types of results:

(i) the fact that it is possible to connect an arbitrary map to a smooth one;

(ii) the fact that the topology of $W^{1,p}(M,N)$ depends only on $[p]$, $M$ and $N$ (and not on the value of $p - [p]$).

These results stimulated a number of important papers, the most notable being the one by Hang and Lin [42].

In the study of Sobolev maps $u$ with values into a manifold $M$, the Jacobian $Ju$ arises naturally [7]. Likewise it plays an essential role in the Ginzburg-Landau equation [43]. It was noted by Ball [4] in a special case, and by Brezis, Coron, and Lieb [27] in general, that the Jacobian $Ju$ computed in the distributional sense detects the singularities of $u$:

$$Ju = \omega_k \sum_{\Gamma \text{ is a } (n-k-1)\text{-manifold of singularities of } u} \deg(u \text{ around } \Gamma) \int_\Gamma d\mathcal{H}^{n-k-1}.$$

In a series of papers, Brezis and collaborators investigated existence and quantitative properties of the Jacobian. In particular, in [15] they proved that the Jacobian extends by continuity from nice maps to arbitrary maps in $W^{s,p}(\Omega;\mathbb{S}^k)$ with $k \leq sp < k+1$. The difficulty behind this result lies in the fact that the Jacobian is given by an integral that does not converge. This is overcome by the introduction of a new extension technique adapted to manifold-valued maps. This new technique proved later to be useful in the study of optimal estimates for the lifting and the degrees [17] and in the study of the Jacobian as a distribution for $C(\mathbb{S}^n;\mathbb{S}^n)$ maps [31]. Further deep work by Brezis and Nguyen on the distributional Jacobian resulted in an illuminating quantitative explanation of Reshetnyak's compactness theorem [47], which among other things is an important tool in the study of elasticity theory. This explanation had not been found before [31] despite thirty-five years of intensive use of Reshetnyak's theorem, especially in elasticity theory!

In connection with uniqueness for lifting, Brezis and collaborators discovered in [13] a characterization of the Sobolev norm in terms of limits of non-local energies. Their result gave a quantitative explanation of the lack of continuity of the scale of $W^{s,p}$-norms as $s \nearrow 1$, and was followed by a number of generalizations, especially from the interpolation and function theory community, e.g. [44, 10]. In particular, the result in [13] suggested the asymptotic behavior of the best constants in limiting Sobolev embeddings which was rigorously established in [14]. These results were extended to other inequalities by many authors. Further developments and extensions were also recently obtained by Brezis and collaborators in [16, 2].

In general terms, Brezis suggested to study more generally characterizations of Sobolev norms and the total variation in terms of limits of non-convex and non-local functionals, compare e.g. [24]. Nguyen's work [45] was a first contribution in this direction. The most far-reaching results in this direction are due to Brezis and Nguyen in [32]. Due to the lack of convexity, the mode of convergence is extremely delicate and many new fascinating and unexpected phenomena as well as pathologies appear. All these results have generated much interest in the image processing community [3] since the functionals associated with these norms are remarkably well adapted for the regularization of non-local functionals that are used for denoising images [1, 38, 39].

Brezis has received considerable recognition for his path breaking research and his scientific leadership.

- He has been elected to membership in 8 national academies (including the French Academy of Science, the American Academy of Arts and Sciences, and the U.S. National Academy of Sciences).
- He holds Doctorates Honoris Causa or Honorary professorships from 11 universities in Europe and Asia.
- He has received prizes from the French Academy of Sciences, the Belgian Academy of Sciences, the College of France, and the A.M.S. (Ky Fan Award).

Brezis' commitment to and work for the mathematical community are also exceptional. Among the most noteworthy examples are the following ones.

- He served as Vice-President of the American Mathematical Society, 2004-2009.
- He is the Chief Editor of the Series on Nonlinear Differential Equations and Applications, Birkhäuser (85 volumes published since 1988).
- He was Chief Editor of Journal of the European Mathematical Society (JEMS) for twelve years (2003-2015), during which time JEMS rose to become a top level journal.
- He is a Member of the Editorial Board of 20 mathematical journals and co-Chief Editor of Comm. Contemp. Math.

In addition to his research achievements, Haim Brezis is known for his extraordinary talent as a teacher and as a speaker. Brezis has a special ability to convey the sense of elegance of the mathematics he is discussing and also brings a genuine enthusiasm that is contagious. This played a role in attracting many young talents to the field. As we already mentioned, Brezis has had remarkable results as a mentor with 57 Ph.D. students. Furthermore, according to the Mathematics Genealogy Project, he has 625 descendants. This sense of elegance is also a reason why the treatise of Brezis on functional analysis [22] is such a successful book and has been translated into many languages. A charismatic speaker at seminars and conferences, he always likes to include fascinating open problems in his lectures. Many of the hundreds of seeds he has thus sown have borne fruit.

As former students of Haim Brezis, and in the name of all of us, we would like to say how much we are indebted to him for his constant scientific support and encouragement. Thank you Haim for your unfailing generosity with time and ideas !

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