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ERRATUM

Volume **186**, Number 2 (2001), in the article "Asymptotics for the Ginzburg–Landau Equation in Arbitrary Dimensions," by F. Bethuel, H. Brezis, and G. Orlandi, pages 432–520 (doi:10.1006/jfan.2001.3791): The proof of Proposition VI.4 on pages 487–489 in Section VI.3 should be modified as follows.

VI.3. Estimates for $|\nabla |u_\varepsilon||$, $1 \leqslant p < 2$

We follow here closely the argument of [Bethuel–Brezis–Hélein 2, Lemma X.13]. Let $1 \leqslant p < 2$ and set

$$\rho = |u_\varepsilon|.$$

The equation for ρ^2 is

$$-\Delta \rho^2 + 2\,|\nabla u|^2 = \frac{2}{\varepsilon^2}\,\rho^2(1-\rho^2) \qquad \text{in } \Omega. \tag{VI.21}$$

We are going to prove

PROPOSITION VI.4. *Let* $1 \leqslant p < 2$. *There exist constants* K_p *and* $0 < \alpha < 1$ *depending only on p,* Ω, K_0, *and* C_0 *such that, for* $0 < \varepsilon < 1$,

$$\int_\Omega |\nabla \rho|^p \leqslant K_0 \varepsilon^\alpha.$$

Proof. We introduce the set

$$S = \{x \in \Omega,\ \rho(x) > 1 - \varepsilon^{1/2}\}$$

and the function

$$\bar\rho = \max\{\rho,\ 1-\varepsilon^{1/2}\},$$

such that $\rho = \bar\rho$ on S and

$$0 \leqslant 1 - \bar\rho \leqslant \varepsilon^{1/2} \qquad \text{in } \Omega. \tag{VI.22}$$

We multiply (VI.21) by $\bar{\rho}^2 - 1$ and integrate over Ω:

$$\int_\Omega \nabla\rho^2\,\nabla\bar{\rho}^2 + \int_\Omega \frac{2\rho^2(1-\rho^2)(1-\bar{\rho}^2)}{\varepsilon^2} = 2\int_\Omega (1-\bar{\rho}^2)\,|\nabla u|^2 - \int_{\partial\Omega} \frac{\partial\rho^2}{\partial n}\,(1-\bar{\rho}^2),$$

$$\int_S |\nabla\rho^2|^2 \leqslant 2\varepsilon^{1/2}\int_\Omega |\nabla u|^2 + 2\int_{\partial\Omega} |\nabla\,|u||\,(1-\bar{\rho}^2).$$

From assumption (H1) we deduce that

$$\int_\Omega |\nabla u|^2 \leqslant K\,|\log\varepsilon|,$$

whereas it follows from (H2) that $1-\bar{\rho}(x) = 0$ if $x \in \partial\Omega$, $\mathrm{dist}(x, \Sigma) \geqslant \varepsilon$, so that

$$\int_{\partial\Omega} |\nabla\,|u||\,(1-\bar{\rho}) \leqslant \varepsilon^{1/2}\int_{\partial\Omega\,\cap\,\mathrm{dist}(x,\,\Sigma)\,\leqslant\,\varepsilon} |\nabla\,|u||$$
$$\leqslant K\varepsilon^{-1/2}\,\mathrm{meas}\{x\in\partial\Omega,\ \mathrm{dist}(x, \Sigma)\leqslant\varepsilon\} \leqslant K\varepsilon^{3/2}.$$

Here we have used the fact that

$$|\nabla u|\,(x) \leqslant \frac{K}{\varepsilon} \qquad \forall x \in \bar{\Omega}.$$

Combining the previous relations we obtain

$$\int_S |\nabla\rho|^2 \leqslant K\varepsilon^{1/2}\,|\log\varepsilon|. \tag{VI.23}$$

Finally, since by (H1)

$$\int_\Omega (1-\rho^2)^2 \leqslant K\varepsilon^2\,|\log\varepsilon|$$

and since $(1-\rho^2) \geqslant \varepsilon^{1/2}$ on $\Omega\backslash S$, we obtain

$$|\Omega\backslash S| \leqslant K\varepsilon\,|\log\varepsilon|.$$

Hence

$$\int_{\Omega\backslash S} |\nabla\rho|^p \leqslant \left(\int_\Omega |\nabla\rho|^2\right)^{p/2} |\Omega\backslash S|^{1-p/2}$$
$$\leqslant K\,|\log\varepsilon|^{p/2}\,|\Omega\backslash S|^{1-p/2}$$
$$\leqslant K\varepsilon^{1-p/2}\,|\log\varepsilon|. \tag{VI.24}$$

Combining (VI.23) with (VI.24) we deduce the result.