Math 555, Fall 2021. Moduli of curves.
Approximate syllabus, to be modified as we go along.

1. Brief review of complex algebraic curves and Riemann surfaces.
- $\mathbb{C}^1$ and $\mathbb{CP}^1$ as schemes and as complex manifolds.
- Elliptic curves.
- Hyperelliptic curves.
- Planar curves

2. Line bundles and divisors on curves.
- Complex-analytic setup and motivation.
- Scheme-theoretic setup.
- $\mathcal{O}(D)$.
- Picard group.
- Regular and rational sections of line bundles.
- Analogy with number fields.

3. Riemann-Roch theorem on curves.
- Reminder of homology and cohomology.
- Canonical class.
- Serre duality.
- Hurwitz formula.

4. Uniformization, Kodaira dimension, automorphisms.
- Genus zero.
- Genus one.
- Higher genera.
- Automorphisms of complex algebraic curves.
- Rough argument for the dimension of the moduli space of curves.

5. Moduli of elliptic curves.
- Complex analytic description.
- Algebraic description, a sniff of DM stacks.
- Level structures and modular curves.
- Monstrous Moonshine.
- (Maybe) Hecke correspondences.

6. Moduli of curves of genus 2.
- Curves of genus 2 and their Jacobians.
- Rough parametrization.
- Igusa quartic.

- Siegel upper half space and Siegel modular forms.
- Various parameterizations.

7. Construction of $M_{g,n}$.

- Tricanonical embedding.
- Hilbert scheme.
- GIT quotient.
- Compactifications.

8. Deligne-Mumford stacks.

- Motivation.
- Category of schemes.
- General stacks.
- Example: finite group quotients.
- Example: BG
- Stack structure on $M_{g,n}$.

9. $M_{0,n}$

10. Kodaira dimension of $M_{g,n}$.

11. Noether-Lefschetz theory.

12. VOAs and Riemann surfaces.

13. Kontsevich's theorem on Airy functon.

14. Gromov-Witten invariants.