Homework 2/17/2020

1. Let R be a Noetherian ring and let S be a multiplicative system in R. Prove that $S^{-1}R$ is also Noetherian.

2. Let (R, m) be a local Noetherian ring. For each positive integer n, prove that m^n/m^{n+1} is a finite-dimensional vector space over R/m.

3. Let M be a finitely generated module over a local ring (R, m). Prove that if $y_1 \bmod m, \ldots, y_k \bmod m$ generate M/mM as an (R/m) vector space, then $y_1, \ldots, y_k$ generate M as an R-module.

4. Let R be a subring of the ring of formal power series in two variables $\mathbb{C}[[x, y]]$ defined as the set of formal power series

$$f = \sum_{i \geq 0, j \geq 0} c_{i,j} x^i y^j$$

with $c_{i,j} = 0$ for all $i > j\sqrt{2}$.

(a) Prove that $m = \{f, \text{ such that } c_{0,0} = 0\}$ is the unique maximum ideal in R.

(b) Prove that m/m^2 is an infinite-dimensional vector space over R/m (which implies R is not Noetherian).