Plan for the course on toric varieties.

September 3, 2020 Affine and projective spaces from different perspectives. (set, variety, scheme). Brief explanation of what schemes are. Difference between variety and variety with embedding.

September 8, 2020 Subvarieties and subschemes of affine and projective spaces. (Examples, including toric examples.)

September 10, 2020 Convex geometry. Convex hull of a set of points, convex polytopes, convex cones. Various technicalities (e.g. support functions). Examples, pictures. Warnings (rigid cones in higher dimensions). Rational polyhedral cones.

September 14, 2020 Affine toric varieties. Definition: semigroup rings, finiteness results. Examples. Normal and non-normal toric varieties.

September 17, 2020 Catching up and discussion of homework exercises.

September 21, 2020 Torus action, from both geometry and algebraic view points. Orbit structure. Identification of the lattice points with characters. Some comments on spherical varieties.

September 24, 2020 Toric variety from a fan: definition. Torus action and orbits.

September 28, 2020 Separatedness. Fan of projective space: example of $\mathbb{CP}^2$. Example: $W\mathbb{P}(1,1,2)$. Smoothness criterion. Compactness criterion.

October 1, 2020 Blowup of a point on $\mathbb{C}^2$. Usual description, toric description, matching up. Toric maps. Birational toric maps.

October 5, 2020 Resolution of toric singularities. Statement of Hironaka's theorem.

October 8, 2020 Catching up and discussion of homework exercises.

October 12, 2020 Closures of torus orbits. Abelian quotient singularities in toric picture. Resolution of abelian quotient singularities in dimension two and continued fractions. Resolutions of A_n singularities, chain of lines.

October 15, 2020 Line bundles. General definition. $\mathcal{O}(1)$ on $\mathbb{P}^n$. Maps to projective space, examples. Segre, Veronese embeddings.

October 19, 2020 Line bundles on toric varieties. Equivariant line bundles. Description in terms of piecewise linear functions. Relation to Weil divisors. Global sections of toric line bundles.

October 22, 2020 Examples, computations. Polarized toric varieties in terms of (moment) polytopes. Dictionary between the two pictures. Example of a singular toric variety with no line bundles, example of a simplicial toric variety with no ample line bundle.

October 26, 2020 Catching up, more examples, exercises.

October 29, 2020 Intersection theory, with focus on surfaces. Definition of intersection numbers on $\mathbb{P}^2$. Bezout's theorem. Intersection theory via deformations and cohomology. Curves with negative self-intersection.

November 2, 2020 Intersection theory on toric surfaces. The resolution of A_n singularities, revisited. Other examples, including Hirzebruch surfaces.

November 5, 2020 Tangent bundle and canonical line bundle. General definitions, then toric case. Euler sequence. Examples. Canonical class in singular case. Fano varieties. Finiteness statements.

November 9, 2020 Proof of finiteness of toric Fano varieties. General discussion of Fano varieties.

November 12, 2020 Toric Mirror symmetry (Batyrev's construction). Reflexive polytopes. Kreuzer-Skarke classification. Reflexive Gorenstein cones. Berglund-Hübsch mirror symmetry.

November 16, 2020 Catching up and homework exercises.

November 19, 2020 Singular cohomology of smooth complex toric varieties. A brief review of algebraic topology and then computation. Prove Stanley-Reisner relations. Prove that they give all in complete case.

November 23, 2020 Cohomology of line bundles on toric varieties. Toric Čech complex. Computing global sections of an ample line bundle on toric variety.

November 30, 2020 Riemann-Roch theorems in general and for toric varieties. Different versions.

December 3, 2020

December 7, 2020

December 10, 2020

Stuff to put in. Toric Mori program. Toric weak factorization. Cox construction and toric DM stacks.