MATH 640:491

Problem Set 1 (Invitation to Mathematical Problem solving.)[1]

We will start working on these problems during the first class session. While many of the problems below ask for short answers, the most important part of the solution is to provide explanation/proof that the answer is correct.

1. How many 0's does 400! end with?
2. If S is a set of real numbers let $S+S$ be the set of all sums of the form $a+b$ where $a \in S$ and $b \in S$ (where a, b are allowed to be the same number). For each positive integer n, how should you choose S of size n if you want to minimize the size of $S+S$?
3. Among all lists of positive integers that sum to 1000, what is the most their product can be?
4. Find all pairs (a, b) of integers such that $a^b = b^a$.
5. Suppose we color the points of the x–y plane with three colors. Prove that there must be two points at distance one from each other that get the same color.
6. Determine all polynomials $P(x)$ that satisfy the equations $P(x^2+1) = (P(x))^2 + 1$ and $P(0) = 0$.
7. Define $a_1 = 1$, and for every $n > 1$, define $a_{n+1} = a_n + \frac{1}{a_n}$. Prove that $19 < a_{200} < 22$.
8. Let us say that a positive real number x is a *Fermat number* if there are three distinct positive integers a, b, c such that $a^x + b^x = c^x$. Prove that there exist arbitrarily large Fermat numbers. (More precisely, show that for every real number M there is a fermat number x that is bigger than M).

[1]Version 6-30-2025