Midterm exam 2, Friday, Nov 8, 2024

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DO NOT OPEN THE EXAM BEFORE THE START ANNOUNCEMENT !

Name:

Each problem is worth 20 points, for a total of 100 points. **Calculators/notes are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** Make sure that I check your name in my list or write your name down. Good luck!

1	2	3	4	5	Total

[1] Let H be an index n subgroup of a group G.

(a)[5pts] Prove that $H_1 = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G.

(b)[15pts] Prove that the index of H_1 in G is at most $n!$.

[1] (20 pts)

Please leave blank!

[2] Let a group G act on a nonempty finite set X. Let n be the number of orbits of this action. Consider the action of G on the set Y of 2-element subsets of X.

(a)[10pts] Prove that the number of orbits of this action of G on Y is at least $\frac{1}{2}n(n-1)$.

(b)[10pts] Suppose the number of orbits of this action of G on Y is exactly $\frac{1}{2}n(n-1)$. Prove that $|X| = n$.

[2] (20 pts)

Please leave blank!

[3] Find all conjugacy classes of the dihedral group D_{10} and write its class equation. (Our convention is that D_{10} has 20 elements.)

[3] (20 pts)

Please leave blank!

[4] Let x be an element of a group G. Denote by $Z(x)$ the centralizer of x. Denote by $N(\langle x\rangle)$ the normalizer of the cyclic subgroup $\langle x\rangle$ generated by x.

(a)[5pts] Prove that $Z(x)$ is a subgroup of $N(\langle x\rangle)$.

(b)[15pts] Prove that $Z(x)$ is a normal subgroup of $N(\langle x\rangle)$.

[4] (20 pts)

Please leave blank!

[5] Prove that all groups of order 2023 are abelian. For your convenience, note that $2023 = 7 \times 289 = 7 \times 17^2$.

[5] (20 pts)

Please leave blank!