Midterm exam 1, Thursday, Oct 4, 2012

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DO NOT OPEN THE EXAM BEFORE THE START ANNOUNCEMENT !

Name:

Each problem is worth 20 points, for a total of 200 points. **Calculators/notes are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** Make sure that I check your name in my list or write your name down. Good luck!

1	2	3	4	5	Total

[1] In which of the following cases is H a subgroup of G? You must give brief justification.

(a) $G = GL_n(\mathbb{C})$ and $H = GL_n(\mathbb{R})$.

(b) $G = \mathbb{R}^\times$ and $H = \{1, -1\}$.

(c) $G = \mathbb{Z}^+$ and H is the set of positive integers.

(d) $G = \mathbb{R}^\times$ and H is the set of positive reals.

(e) $G = GL_2(\mathbb{R})$ and H is the set of matrices $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $a \neq 0$.

[1] (20 pts)

Please leave blank!

[2] (a) Prove that, for $n \geq 2$, transpositions generate the symmetric group S_n.

(b) Prove that, for $n \geq 3$, the three-cycles generate the alternating group A_n.

[2] (20 pts)

Please leave blank!

[3] Let G be a cyclic group of order 30.

(a) How many subgroups does G have? How many of these are normal subgroups?

(b) If $\phi : G \to G'$ is a group homomorphism and G' has order 35, what are possible groups $Ker(\phi)$ and $Im(\phi)$?

(c) Show that the automorphism group of G is isomorphic to $(\mathbb{Z}/30\mathbb{Z})^{\times}$.

[3] (20 pts)

Please leave blank!

[4] (a) Show that if a subgroup H of a group G is not normal, there are left cosets aH and bH whose product is not a coset.

(b) Illustrate the statement of part (a) by an example of your choice.

[4] (20 pts)

Please leave blank!

[5] Let G be a group and H be its subgroup. Consider the subset N in G which consists of all elements $a \in G$ such that the left coset aH is equal to the right coset Ha.

(a) Prove that N is a subgroup of G.

(b) Prove that H is a normal subgroup of N.

(c) Consider $G = GL_2(\mathbb{R})$, and H the subgroup of matrices $\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}$, $b \in \mathbb{R}$. Calculate N.

[5] (20 pts)

Please leave blank!