Midterm exam 2, Thursday, Nov 11, 2022

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DO NOT OPEN THE EXAM BEFORE THE START ANNOUNCEMENT !

Name:

Each problem is worth 20 points, for a total of 100 points. **Calculators/notes are not allowed on this test.** Please read each question carefully, it also helps to check afterwards that you have answered each part of each question. **You must show all your work to receive credit.** Make sure that I check your name in my list or write your name down. Good luck!

1	2	3	4	5	Total

[1] Consider the action of the group $G = D_4$ of symmetries of the square $ABCD$ on the set of its vertices $U = \{A, B, C, D\}$. Compute the orbits of the diagonal action of G on the set $U \times U$, defined as $g(x, y) = (gx, gy)$.

[1] (20 pts)

Please leave blank!

[2] Suppose that a finite group G acts on a finite set S. For each $g \in G$ define by S^g the set of $x \in S$ such that $gx = x$ (i.e. the fixed point set of g).

(a)[14pts] Prove that if the action is transitive, then $|G| = \sum_{g \in G} |S^g|$.

(b)[6pts] Prove that for any action the number of orbits is equal to $\frac{1}{|G|} \sum_{g \in G} |S^g|$.

[2] (20 pts)

Please leave blank!

[3] Let H be an index 2 subgroup of a finite group G. Let C be a conjugacy class in G. Prove that $C \cap H$ is either a conjugacy class in H or is a disjoint union of two conjugacy classes C_1 and C_2 of H with $|C_1| = |C_2|$.

In-exam correction. Need to assume $C \cap H \neq \emptyset$.

[3] (20 pts)

Please leave blank!

[4] Let G be a nonabelian group of 125 elements. Prove that G has exactly 29 different conjugacy classes.

[4] (20 pts)

Please leave blank!

[5] Let G be a group of order $2pq$ where $p < q$ are odd primes. Prove that G is not simple.

[5] (20 pts)

Please leave blank!