Problem 1. Let S be a set of cardinality (size) n. What is the cardinality of the set of all subsets of S?

Problem 2. (a) If the function $f : S_1 \to S_2$ is one-to-one and $g : S_2 \to S_3$ is one-to-one, is it guaranteed that $g \circ f : S_1 \to S_3$ is also one-to-one?

(b) If $f : S_1 \to S_2$ and $g : S_2 \to S_3$ are such that f and $g \circ f$ are one-to-one, is it guaranteed that g is one-to-one?

You must justify you answers, which means that if you claim that the conclusion is guaranteed, then you need to prove it. If you claim that the conclusion is not guaranteed, then you need to provide a counterexample.

Problem 3. Show that the set of all real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers is a field with respect to the usual operations of addition and multiplication.

Problem 4. Give an example of a field of characteristic 2.

Problem 5. (a) Let F be a field of characteristic 2. Prove that for any elements x and y of F there holds

$$(x + y)^2 = x^2 + y^2.$$

(b) Let F be a field of characteristic not equal to 2. Prove that if for $x, y \in F$ there holds

$$(x + y)^2 = x^2 + y^2$$

then one or both of x and y are zero.

Problem 6. Calculate the following complex numbers (i.e. write them in the form $a + bi$ where a and b are real):

$$\frac{2 + 3i}{4 - 5i}; \quad (2 + 3i)^2; \quad \overline{2 + 3i}$$