Calculus 251:C3 Worksheet 16.8

(1) Verify the Divergence Theorem for the given vector field and region.

(a) $\vec{F}(x, y, z) = \langle z, x, y \rangle$, the box $[0, 4] \times [0, 2] \times [0, 3]$.

(b) $\vec{F}(x, y, z) = \langle y, x, z \rangle$, the region $x^2 + y^2 + z^2 \leq 4$.

(c) $\vec{F}(x, y, z) = \langle 2x, 3z, 3y \rangle$, the region $x^2 + y^2 \leq 1, 0 \leq z \leq 2$.

(2) Use the Divergence Theorem to evaluate the flux $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.

(a) $\vec{F}(x, y, z) = \langle xy^2, yz^2, zx^2 \rangle$
$\mathcal{S}$ is the boundary of the cylinder $x^2 + y^2 \leq 4, 0 \leq z \leq 3$

(b) $\vec{F}(x, y, z) = \langle x^3, 0, z^3 \rangle$
$\mathcal{S}$ is the boundary of the region $x^2 + y^2 + z^2 \leq 4$, $x, y, z \geq 0$

(c) $\vec{F}(x, y, z) = \langle x, y^2, z + y \rangle$
$\mathcal{S}$ is the boundary of the region contained in the cylinder $x^2 + y^2 = 4$ between the planes $z = x$ and $z = 8$

(d) $\vec{F}(x, y, z) = \langle x^2 - z^2, e^{z^2} - \cos x, y^3 \rangle$
$\mathcal{S}$ is the boundary of the region bounded by $x + 2y + 4z = 12$ and the coordinate planes in the first octant

(e) $\vec{F}(x, y, z) = \langle x + y, z, z - x \rangle$
$\mathcal{S}$ is the boundary of the region between the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane