Calculus 251:C3 Worksheet 16.7

(1) Verify Stokes' Theorem for the given vector field and surface, orienting the surface with upward-pointing normal.

(a) $\vec{F} = \langle 2xy, x, y+z \rangle$, the surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$.

(b) $\vec{F} = \langle y, x, x^2 + y^2 \rangle$, the surface $x^2 + y^2 + z^2 = 1$ for $z \geq 0$.

(2) Caculate curl$(\vec{F})$ and then apply Stokes' Theorem to compute the flux of curl$(\vec{F})$ through the given surface using a line integral.

(a) $\vec{F} = \left\langle e^{z^2} - y, e^{z^3} + x, \cos(xz) \right\rangle$, the surface $x^2 + y^2 + z^2 = 1$ for $z \geq 0$ with outward-pointing normal.

(b) $\vec{F} = \langle 3z, 5x, -2y \rangle$, the surface $z = x^2 + y^2$ for $z \leq 4$ with upward-pointing normal.

(c) $\vec{F} = \langle yz, xz, xy \rangle$, the surface $x^2 + y^2 = 1$ for $1 \leq z \leq 4$ with outward-pointing normal.

(3) Use Stokes' Theorem to evaluate $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ by finding the flux of curl$(\vec{F})$ across an appropriate surface.

(a) $\vec{F} = \langle 3y, -2x, 3y \rangle$, $\mathcal{C}$ is the circle $x^2 + y^2 = 9, z = 2$ oriented counterclockwise as viewed from above.

(b) $\vec{F} = \langle xz, xy, yz \rangle$, $\mathcal{C}$ is the rectangle with vertices $(0,0,0), (0,0,2), (3,0,2), (3,0,0)$ oriented counterclockwise as viewed from the positive y-axis.

(c) $\vec{F} = \langle y, z, x \rangle$, $\mathcal{C}$ is the rectangle with vertices $(0,0,0), (3,0,0), (0,3,3)$ oriented counterclockwise as viewed from above.

(4) Let $\vec{F} = \langle y^2, 2z + x, 2y^2 \rangle$. Use Stokes' Theorem to find a plane with equation $ax + by + cz = 0$ (where a, b, c are not all zero) such that $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r} = 0$ for every closed $\mathcal{C}$ lying in the plane.

(5) Let $\vec{F} = \langle y^2, x^2, z^2 \rangle$. Show that for any two closed curves $\mathcal{C}_1, \mathcal{C}_2$ going exactly once around a cylinder whose central axis is the z-axis, $\oint_{\mathcal{C}_1} \vec{F} \cdot d\vec{r} = \oint_{\mathcal{C}_2} \vec{F} \cdot d\vec{r}$

(6) Let I be the flux of $\vec{F} = \left\langle e^y, 2xe^{x^2}, z^2 \right\rangle$ through the upper hemisphere $\mathcal{S}$ of the unit sphere.

(a) Let $\vec{G} = \left\langle e^y, 2xe^{x^2}, 0 \right\rangle$. Find a vector field $\vec{A}$ such that $\vec{\nabla} \times \vec{A} = \vec{G}$.

(b) Use Stokes' Theorem to show that the flux of $\vec{G}$ through $\mathcal{S}$ is 0. (*Hint:* Calculate the circulation of $\vec{A}$ around $\partial\mathcal{S}$.)

(c) Calculate I. (*Hint:* Use (b) to show that I is the flux of $\langle 0, 0, z^2 \rangle$ through $\mathcal{S}$.)