Calculus 251:C3 Worksheet 16.6

(1) Let $\mathcal{S}$ be the triangle with vertices $(0, 0, 3)$, $(1, 0, 2)$, and $(0, 4, 1)$.

(a) Find a parametrization for $\mathcal{S}$.

(b) Calculate $\iint_{\mathcal{S}} (xy + e^z)\, dS$

(2) Let $\mathcal{S}$ be the portion of the graph of$z = x + y^2$ above the triangle in the xy-plane with vertices $(0, 0, 0)$, $(1, 1, 0)$, and $(0, 1, 0)$. Calculate $\iint_{\mathcal{S}} (z - x)\, dS$

(3) For each vector field $\vec{F}$ and oriented surface $\mathcal{S}$, calculate the flux integral $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$

(a) $\vec{F} = \langle e^z, z, x\rangle$; $\mathcal{S}$ is parametrized by $G(u, v) = (uv, u + v, u)$ on the domain $[0, 1] \times [0, 1]$

(b) $\vec{F} = \langle x, x, y\rangle$; $\mathcal{S}$ is the triangle with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$ and oriented upward.

(c) $\vec{F} = \langle z^2, x, -3z\rangle$; $\mathcal{S}$ is the portion of the graph of $z = 4 - y^2$ cut by the planes $x = 0$, $x = 1$, and $z = 0$, and oriented by a normal vector pointing away from the x-axis

(d) $\vec{F} = \langle -x, -y, z^2\rangle$; $\mathcal{S}$ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$, and oriented by a normal vector pointing away from the z-axis

(e) $\vec{F} = \langle xy, yz, 1\rangle$; $\mathcal{S}$ is the upper cap of the sphere $x^2 + y^2 + z^2 = 25$ cut by the plane $z = 3$, and oriented inward

(f) $\vec{F} = \langle 2xy, 2yz, 2xz\rangle$; $\mathcal{S}$ is the boundary of the cube $[0, a] \times [0, b] \times [0, c]$ with $a, b, c > 0$, and oriented outward

(g) $\vec{F} = \langle 4x, 4y, 2\rangle$; $\mathcal{S}$ is the bottom portion of the paraboloid $z = x^2 + y^2$ cut by the plane $z = 3$, and oriented downward