

Calculus 251:C3      Worksheet 16.5

- (1) Let  $\mathcal{S}$  be the surface with parametrization  $G(u, v) = (u^2 - v^2, u + v, u - v)$ . Find an equation of the plane tangent to  $\mathcal{S}$  at the point where  $u = 2$  and  $v = 3$ .
- (2) Let  $\mathcal{S}$  be the sphere centered at the origin with radius 3. Find an equation of the plane tangent to  $\mathcal{S}$  at the point  $(x, y, z) = (1, -2, 2)$ .
- (3) Calculate the area of the portion of the plane  $y + 2z = 2$  inside the cylinder  $x^2 + y^2 = 1$ .
- (4) Calculate the area of the lower portion cut from the sphere  $x^2 + y^2 + z^2 = 2$  by the cone  $z = \sqrt{x^2 + y^2}$ .
- (5) A surface is parametrized by  $G(x, y) = (x, y, xy)$ . Let  $\mathcal{S}$  be the part of the surface with parameter domain  $\mathcal{D} = \{(x, y) : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$ .
  - (a) Calculate  $\vec{T}_x$ ,  $\vec{T}_y$ , and  $\vec{N}(x, y)$ .
  - (b) Calculate the surface area of  $\mathcal{S}$ .
  - (c) Technically a problem for §16.6: calculate the average  $z$ -coordinate of  $\mathcal{S}$ .
- (6) Let  $\mathcal{S}$  be the surface that consists of the portion of the sphere  $x^2 + y^2 + z^2 = 9$  in the first octant that is also both outside the cylinder  $x^2 + y^2 = 1$  and above the plane  $z = \sqrt{5}$ .
  - (a) Find a parametrization for  $\mathcal{S}$ .
  - (b) Calculate the normal vector  $\vec{N}$ . Does this normal vector point inward or outward?