Calculus 251:C3 Worksheet 16.4

(1) Let $\mathcal{C}$ be the piecewise linear path from $(1,0)$ to $(0,1)$ to $(-1,0)$ to $(1,0)$. Verify Green's Theorem (Circulation-Curl Form) for the line integral $\displaystyle\int_{\mathcal{C}}(xy\ dx + (x^2+x)\ dy$.

(2) Use Green's Theorem to calculate $\displaystyle\int_{\mathcal{C}}(y^2\ dx + x^2\ dy)$ where $\mathcal{C}$ is the triangular path from $(0,0)$ to $(0,1)$ to $(1,0)$ to $(0,0)$.

(3) Let $\mathcal{C}$ be the boundary of the region bounded by $y = x^2$ and $x = y^2$. Assume that $\mathcal{C}$ is oriented counterclockwise. Calculate $\displaystyle\int_{\mathcal{C}}\big((xy+y^2)dx + (x-y)dy\big)$.

(4) Consider the vector field $\vec{F} = \dfrac{1}{r}\vec{e}_r$. (Reminder: in $\mathbb{R}^2$, $\vec{e}_r = \left\langle \dfrac{x}{\sqrt{x^2+y^2}}, \dfrac{y}{\sqrt{x^2+y^2}} \right\rangle$

(a) Let $\mathcal{C}$ be the circle of radius R centered at the origin, oriented counterclockwise. Evaluate the integral of $\vec{F}$ along $\mathcal{C}$ directly (no fancy theorems).

(b) Calculate $\vec{\nabla} \times \vec{F}$.

(c) Does $\vec{F}$ have a potential? If so, calculate a potential for $\vec{F}$.

(d) Recall that the integral of a conservative vector field along a closed curve is 0. Does this contradict your answers to parts (a), (b), and (c)?

(e) Repeat parts (a), (b), and (c) with the vector field $\vec{F} = \dfrac{1}{r^2}\langle -y, x\rangle$.

(f) Part (a) asks you to evaluate all line integrals in this problem directly without any fancy theorems. Now consider using Green's theorem to solve this problem. What can you say?

(5) The velocity field of a certain fluid is given by $\vec{u} = \left\langle e^x \ln(y), x^2 + \frac{e^x}{y} \right\rangle$. Calculate the circulation of the fluid around the boundary of the region that is bounded above by $y = 3 - x$ and bounded below by $y = x^2 + 1$. Assume the boundary is oriented counterclockwise.