Difficulty guide for worksheet:	
C-level or B-level exam problem:	1, 2, 3, 4
A-level exam problem or challenge for extra study:	5
beyond the scope and/or removed from syllabus:	none

1. Suppose the vector field $\boldsymbol{F}$ is continuous and $\boldsymbol{F} = \langle f, g \rangle = \boldsymbol{\nabla} u$, with $u(1,2) = 7$, $u(3,6) = 10$, and $u(6,4) = 20$. Evaluate the following integrals, if possible.

(a) $\displaystyle\int_{\mathcal{C}} \boldsymbol{F} \cdot d\boldsymbol{r}$, where $\mathcal{C}$ is the path $\boldsymbol{r}(t) = \langle 2t-1,\ t^2+t \rangle$ for $1 \le t \le 2$

(b) $\displaystyle\int_{\mathcal{C}} \boldsymbol{F} \cdot \boldsymbol{T}\, ds$, where $\mathcal{C}$ is a smooth curve from $(1,2)$ to $(6,4)$

(c) $\displaystyle\int_{\mathcal{C}} (f\, dx + g\, dy)$, where $\mathcal{C}$ is the path consisting of the line segment from $(6,4)$ to $(1,2)$ followed by the line segment from $(1,2)$ to $(3,6)$

(d) $\displaystyle\int_{\mathcal{C}} \boldsymbol{F} \cdot d\boldsymbol{r}$, where $\mathcal{C}$ is a circle, oriented clockwise, starting and ending at $(7,3)$

2. Let $\boldsymbol{F} = \langle \cos(2y-z),\ -2x\sin(2y-z),\ x\sin(2y-z) \rangle$.

(a) Show that $\boldsymbol{F}$ is conservative on $\mathbb{R}^3$.

(b) Find a potential for $\boldsymbol{F}$.

(c) Let $\mathcal{C}$ be the path $\boldsymbol{r}(t) = \langle t^2, t, t \rangle$ for $0 \le t \le \pi$. Calculate the line integral of $\boldsymbol{F}$ along $\mathcal{C}$.

3. Calculate the work done by the force $\boldsymbol{F} = e^{-x} \langle \cos(y),\ \sin(y) \rangle$ on a particle that travels around the square $[-1,1] \times [-1,1]$ anticlockwise, starting from the upper right vertex.

4. Let $\mathcal{C}$ be the path $\boldsymbol{r}(t) = \langle t^2,\ t^3,\ t-1 \rangle$ for $1 \le t \le 2$. Calculate the line integral

$$\int_{\mathcal{C}} (ye^z\, dx + xe^z\, dy + xye^z\, dz)$$

5. Let $\boldsymbol{F} = \langle 2xy+5,\ x^2-4z,\ -3y \rangle$ and $\boldsymbol{G} = \langle 0,\ 0,\ y \rangle$, and put $\boldsymbol{H} = \boldsymbol{F} - \boldsymbol{G}$. Let $\mathcal{C}$ be the path $\boldsymbol{r}(t) = \langle e^t \sin(\pi t^3/2),\ e^{-t}\cos(\pi t), e^t \rangle$ for $0 \le t \le 1$.

(a) Show that both $\boldsymbol{F}$ and $\boldsymbol{G}$ are not conservative, but $\boldsymbol{H}$ is conservative.

(b) Find a potential for $\boldsymbol{H}$.

(c) Use part (b) to calculate the line integral of $\boldsymbol{H}$ along $\mathcal{C}$.

(d) Calculate the line integral of $\boldsymbol{G}$ along $\mathcal{C}$.

(e) Use parts (c) and (d) to calculate the line integral of $\boldsymbol{F}$ along $\mathcal{C}$.