Calculus 251:C3 Worksheet 16.1

(1) For each part, calculate $\int_{\mathcal{C}} f\, ds$ for the given function f and curve $\mathcal{C}$.

(a) $f(x,y) = xy$
$\mathcal{C}$ is the unit circle

(b) $f(x,y) = x^2 - 2y^2$
$\mathcal{C}$ is the line segment from the origin to the point $(\sqrt{8}, \sqrt{8})$

(c) $f(x,y) = x$
$\mathcal{C}$ is the curve $\vec{r}(t) = \langle t^3, 4t\rangle$, for $0 \le t \le 1$

(d) $f(x,y,z) = y - z$
$\mathcal{C}$ is one turn of the helix $\vec{r}(t) = \langle 3\cos(t), 3\sin(t), 4t\rangle$ starting from $(3,0,0)$

(e) $f(x,y,z) = xz$
$\mathcal{C}$ is the line segment from the origin to $(7,9,10)$

(f) $f(x,y,z) = xz$
$\mathcal{C}$ is the line segment from the origin to $(3,2,6)$ followed by the line segment from $(3,2,6)$ to $(7,9,10)$

(g) $f(x,y,z) = xz$
$\mathcal{C}$ is the line segment from the origin to $(7,0,0)$ followed by the line segment from $(7,0,0)$ to $(7,9,10)$

(h) $f(x,y,z) = xz$
$\mathcal{C}$ is the line segment from the origin to $(7,0,10)$ followed by the line segment from $(7,0,10)$ to $(7,9,10)$

(2) Let $\mathcal{C}$ be the curve $y = x^{4/3}$ in the xy-plane from $(1,1)$ to $(8,16)$. Let $\mathcal{R}$ be the sheet that consists of all points below the surface $z = x/y$ and above the curve $\mathcal{C}$. Calculate the area of $\mathcal{R}$

(3) Find the mass of a wire that lies along the curve $\vec{r}(t) = (t^2-1)\hat{\mathbf{j}} + 2t\hat{\mathbf{k}}$, $0 \le t \le 1$, if the density is...

(a) $\delta = 4$

(b) $\delta(t) = \frac{3t}{2}$

(c) $\delta(x,y,z) = z$

(d) $\delta(x,y,z) = y + 2$