

Difficulty guide for worksheet:

C-level or B-level exam problem: 1a, 1b, 1c, 1e, 2a, 3, 4, 5

A-level exam problem or challenge for extra study: 1d, 1f, 2b, 6

beyond the scope and/or removed from syllabus: none

1. Find the volume of each region or type of solid.

- (a) the region bounded by the paraboloid $z = x^2 + y^2$ and the cone $z = 2 - \sqrt{x^2 + y^2}$
- (b) the region below the surface $z = xy + 10$ and above the annular region $\mathcal{R} = \{(x, y) : 4 \leq x^2 + y^2 \leq 16\}$
- (c) the region inside both the cone $\varphi = \frac{\pi}{6}$ and the sphere $\rho = 4$
- (d) a spherical cap of radius R and height H
- (e) the solid obtained from a sphere centered at the origin with radius 2 after a cylindrical hole of radius 1 is drilled through the center of the sphere perpendicular to its base
- (f) the solid bounded by the cylinder $x^2 + y^2 = 1$, the xy -plane, and the plane $z = x + y$

4. Calculate $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_0^{9-3\sqrt{x^2+y^2}} dz dx dy$ by changing to cylindrical coordinates.

5. Let \mathcal{W} be the solid region bounded above by the plane $z = 5$ and bounded below by the cone $z^2 = x^2 + y^2$. Use spherical coordinates to calculate $\iiint_{\mathcal{W}} \sqrt{x^2 + y^2 + z^2} dV$.

6. Let \mathcal{W} be the region within the cylinder $x^2 + y^2 = 2$ between the xy -plane and the cone $z = \sqrt{x^2 + y^2}$. Calculate the integral of $f(x, y) = x^2 + y^2$ over \mathcal{W} using...

- (a) ...rectangular coordinates.
- (b) ...cylindrical coordinates.
- (c) ...spherical coordinates.

Which of these do you think is easiest?