Calculus 251:C3   Worksheet 15.3-15.4

(1) Sketch the region bounded by the given lines and curves, then write and evaluate a double integral to find the area.

    (a) The x-axis, the y-axis, and the line $4x + 7y = 28$

    (b) The parabola $y = -x^2$ and the line $y = x + 2$

    (c) The parabolas $y = x^2$ and $y = 2x - x^2$

    (d) The lines $y = x$, $y = \frac{1}{3}x$, and $y = 4$

    (e) The lines $y = x - 2$ and $y = -x$ and the curve $y = \sqrt{x}$

    (f) The smaller region bounded by the circle $x^2 + y^2 = 4$ and the line $y = 2 - x$

(2) Find the average height of the paraboloid $z = x^2 + y^2$ above the square $[0, 2] \times [0, 2]$.

(3) Find the average value of $f(x, y) = \sin(x + y)$ over each of the following regions:

    (a) The rectangle $[0, \pi] \times [0, \pi]$

    (b) The rectangle $[0, \pi] \times [0, \pi/2]$

    (c) The triangle bounded by the coordinate axes and the line $y = \pi - x$

(4) Evaluate $\displaystyle\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} dy\, dx$. Yes, you should convert to polar coordinates first!

(5) Find the area of the region inside the circle $x^2 + y^2 = 4x$.

(6) Evaluate $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1 + x^2 + y^2)^2}\, dy\, dx$.

(7) Find the average height of the cone $z = \sqrt{x^2 + y^2}$ above the disk $x^2 + y^2 \leq a^2$ in the xy-plane.

(8) Find the average distance from a point $P = (x, y)$ in the disk $x^2 + y^2 \leq a^2$ to the origin.