Calculus 251:C3 Worksheet 15.1-15.2

(1) Evaluate each integral.

(a) $\displaystyle\int_1^3 \int_0^2 x^3 y \, dydx$

(b) $\displaystyle\int_4^9 \int_{-2}^{10} (-5) \, dydx$

(c) $\displaystyle\int_0^4 \int_0^9 \sqrt{2x+8y} \, dydx$

(d) $\displaystyle\int_1^2 \int_{-1}^2 e^{3x-2y} \, dydx$

(e) $\displaystyle\int_0^4 \int_0^5 \frac{1}{\sqrt{x+y}} \, dydx$

(f) $\displaystyle\int_0^1 \int_0^{\pi/2} y\cos^3(xy) \, dydx$

(2) Calculate the volume under the graph of $z = x\ln(y)$ and above the rectangle in the xy-plane with lower left vertex $(0,1)$ and upper right vertex $(2, e^3)$.

(3) Let $\mathcal{W}$ be the solid region below the graph of $z = y\sqrt{1+xy}$ and above the rectangle $[1,4] \times [1,9]$ in the xy-plane. Calculate the volume of $\mathcal{W}$ using...

(a) ...an integral in the order $dydx$.

(b) ...an integral in the order $dxdy$.

Which integral is easier?

(4) Calculate $\displaystyle\iint_{\mathcal{D}} y \, dA$, where $\mathcal{D}$ is the domain

$$\mathcal{D} = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 1, x^2 \le y \le 4 - x^2\}$$

(5) Let $\mathcal{T}$ be the triangle in the xy-plane with vertices $(3,0)$, $(0,6)$, and $(3,6)$. Calculate $\displaystyle\iint_{\mathcal{T}} xy \, dA$.

(6) Let $\mathcal{T}$ be the trapezoid in the xy-plane with vertices $(0,0)$, $(4,0)$, $(4,2)$, and $(2,2)$. Suppose $f(x,y)$ is continuous on $\mathcal{T}$. Write $\displaystyle\iint_{\mathcal{T}} f(x,y) \, dA$ as an iterated integral, or sum of iterated integrals, in each possible order ($dydx$ and $dxdy$).

(7) Let $\mathcal{R}$ be the smaller of the two regions in the xy-plane bounded by $x^2 + y^2 = 4$ and $x = 1$. Calculate $\displaystyle\iint_{\mathcal{R}} \frac{y}{x} \, dA$.

(8) For each integral, sketch the domain of integration and express as an iterated integral in the opposite order. For parts (c) and (d), also evaluate the integral.

(a) $\displaystyle\int_0^8 \int_x^8 f(x,y) \, dydx$

(b) $\displaystyle\int_0^1 \int_{e^x}^e f(x,y) \, dydx$

(c) $\displaystyle\int_0^1 \int_y^1 \frac{\sin x}{x} \, dxdy$

(d) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1} \, dxdy$