Difficulty guide for worksheet:	
C-level or B-level exam problem:	1a, 1b, 1c, 1d, 1f, 1h, 1j, 2b, 3a
A-level exam problem or challenge for extra study:	1e, 1g, 1i, 2a
beyond the scope and/or removed from syllabus:	1k, 1l, 3b

1. Use the method of Lagrange multipliers to find the described (constrained) extreme value. Examine the constraints carefully. You may also need techniques from section 14.7 (finding interior critical points). There also may be more than one constraint.
Note: It may also be the case that a particular extreme value does not exist. If this is the case, indicate which extreme value (minimum or maximum or both) does not exist and explain why.

(a) Find the minimum and maximum values of $f(x, y) = x^2 + y^2 - xy - 4$ subject to $x + y = 6$.

(b) Maximize $f(x, y) = x^2 - 2y - y^2$ subject to $x^2 + y^2 \leq 1$.

(c) Find the point on the ellipsoid $4x^2 + 2y^2 + z^2 = 4$ that is closest to the origin.

(d) Find the point on the ellipse $x^2 + 6y^2 + 3xy = 40$ with the smallest y-coordinate.

(e) Maximize $f(x, y) = \ln(xy^2)$ subject to $2x^2 + 3y^2 = 8$ with $x > 0$ and $y > 0$.

(f) Find the points on the curve $xy = 4$ with the smallest and largest values of $xy^2 + x + y$.

(g) Find the smallest and largest values of $f(x, y, z) = 2x^2 + 4y^2 + z^2$ subject to $4x - 8y + 2z = 10$.

(h) Maximize $f(x, y, z) = x^2y^2z^2$ on the sphere $x^2 + y^2 + z^2 = R^2$. (Note that R is a constant.)

(i) Find the point on the plane $2x + y + z = 1$ that is closest to the origin.

(j) The temperature T at point (x, y, z) is $T = 200 - xy - xz - yz$. Find the lowest temperature on the plane $x + y + z = 10$.

(k) Find the minimum of $f(x, y, z) = x^2 + z^2 + z^2$ subject to $x + y = 4$ and $y + z = 6$.

(l) Find the maximum of $f(x, y, z) = xyz$ subject to $x^2 + y^2 = 3$ and $y = 2z$.

2. For each part, use the method of Lagrange multipliers to solve the optimization problem.

(a) Find the volume of the largest rectangular prism that can be inscribed in the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

(b) A manufacturer has \$8,000 to spend on the development and promotion of a new product. It is estimated that if x thousand dollars is spent on development and y thousand is spent on promotion, sales will be approximately

$$f(x, y) = 50x^{1/2}y^{3/2}$$

units. How much money should the manufacturer allocate to development and how much to promotion to maximize sales?

3. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, the output at a certain factor may be modeled by

$$Q(x, y) = 60x^{1/3}y^{2/3}$$

units. Assume \$120,000 are available.

(a) How should money be allocated between labor and equipment to generate the largest possible output?

(b) Use the Lagrange multiplier λ to estimate the change in the maximum output of the factory that would result if the money available for labor and equipment is increased by \$1,000.