Calculus 251:C3   Worksheet 14.3-14.4

(1) Calculate all first derivatives for each function f.

(a) $f(x,y) = \cos\left(\dfrac{y}{x+y}\right)$

(b) $f(u,v) = \ln(u^2 + uv)$

(c) $f(x,y,z) = ze^{xz - x^2z^3}$

(d) $f(s,t) = \tan^{-1}(st^2)$

(2) Calculate f_{xyxzy} for the following function:

$$f(x,y,z) = y\sin(xz)\sin(x+z) + (x+z^2)\tan(y) + x\tan\left(\frac{z+z^{-1}}{y-y^{-1}}\right)$$

(3) Prove that there is no function f such that $f_x = xy^2$ and $f_y = -x^2y$

(4) Let r, s, and t be independent parameters and suppose x, y, and z are given by

$$x = 4 - s - 6t$$

$$y = -r + 2s + 5t$$

$$z = 7r + 3s - t$$

Let $w = f(x,y,z)$ where f is an arbitrary differentiable function. Calculate the sum

$$A(r,s,t) = \frac{\partial w}{\partial r} - 2\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t}$$

Write your answer as a function of r, s, and t. Simplify as much as possible. Since f is arbitrary, your answer may still contain the symbol f or related symbols. But you must write your answer as a function of r, s, and t.

(5) Let $z = f(x,y)$, where f is an arbitrary differentiable function. Suppose $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Calculate both z_r and z_θ. Express your answer in terms of r and θ only.
This exercise shows how the derivatives in polar coordinates are related to the derivatives in rectangular coordinates.

(6) Use the multivariable chain rule to calculate the specified partial derivatives.

(a) z_s and z_t, where $z = x^2\sin(y)$, $x = s - t$, and $y = t^2$

(b) w_s and w_t, where $w = \frac{x-z}{y+z}$, $x = s + t$, $y = st$, and $z = s - t$

(c) $\frac{dU}{dt}$, where $U = \frac{xy^2}{z^8}$, $x = e^t$, $y = \sin(3t)$, and $z = 4t + 1$

(7) Suppose x and y are implicitly related by the equation $F(x,y)=0$. Use the multivariable chain rule to show

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y}$$

Then use this result to find $y'(x)$ if $x^2 = xy^2 + \sin(y) + 3$.

(8) . Suppose x, y, and z are implicitly related by the equation $F(x,y,z)=0$. Use the multivariable chain rule to show

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Then use this result to find z_x and z_y if $x^2yz^2 = 10 - 3xz^3$.

(9) For each implicit equation, calculate the specified partial derivative.

(a) z_x if $\sqrt{x^2 + 2xz + z^4} = 3$

(b) y_z if $y\ln(x^2 + y^2 + 4z) = 1$