Difficulty guide for worksheet:

C-level or B-level exam problem:	1a, 1b, 1c, 1d, 1f, 2, 3a, 3b, 3c
A-level exam problem or challenge for extra study:	1e, 1g, 1h, 3d, 3e, 3f
beyond the scope and/or removed from syllabus:	4

1. For each function f, describe the contour curves or contour surfaces, as appropriate. You should give a complete, concise, and clear English description in addition to a sketch of a contour map with an appropriate contour interval.

(a) $f(x,y) = 3x^2 - y^2$

(b) $f(x,y) = \dfrac{y}{x^2}$

(c) $f(x,y) = x^4$

(d) $f(x,y) = e^{x^2+4y^2}$

(e) $f(x,y) = \sin(x^2+y^2)$

(f) $f(x,y,z) = 2x - 3y + 4z - 5$

(g) $f(x,y,z) = x^2 + y^2 - z$

(h) $f(x,y,z) = x^2 - y^2 + z^2$

2. Sketch a graph of the function $f(x,y) = \sqrt{9 - x^2 - y^2}$.

3. Evaluate the limit or determine it does not exist. You must justify your answer.

(a) $\displaystyle\lim_{(x,y)\to(1,1)} \left(\frac{e^x - e^{-y}}{x+y}\right)$

(b) $\displaystyle\lim_{(x,y)\to(1,0)} \ln(x^2 - y)$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{xy}{x^2+y^2}\right)$

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{x^3+y^3}{xy^2}\right)$

(e) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{x^3}{x^2+y^2}\right)$

(f) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\tan(x)\sin\left(\frac{1}{|x|+|y|}\right)\right)$

4. Suppose two distinct contour curves of the function $f(x,y)$ are tangent to each other at the point $P = (a,b)$. What can you say about $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y)$? Explain your answer.

Solutions:

1. (a) hyperbolas with focus and directrix on coordinate axes

   (b) parabolas through the origin with origin removed

   (c) vertical lines in right half-plane

   (d) ellipses centered at origin

   (e) concentric circles centered at origin.

   (f) planes

   (g) paraboloids opening along z-axis

   (h) cones opening along x-axis

2. upper hemisphere of sphere centered at origin with radius 3.

3. (a) $\dfrac{e - e^{-1}}{2} = \sinh(1)$

   (b) 0

(c) DNE

(d) DNE

(e) 0

(f) 0

4 $\lim_{(x,y)\to(a,b)} f(x,y)$ DNE