

**Difficulty guide for worksheet:**

*C-level or B-level exam problem:* 1a, 1b, 1c, 1d, 1f, 2, 3a, 3b, 3c

*A-level exam problem or challenge for extra study:* 1e, 1g, 1h, 3d, 3e, 3f

*beyond the scope and/or removed from syllabus:* 4

1. For each function  $f$ , describe the contour curves or contour surfaces, as appropriate. You should give a complete, concise, and clear English description in addition to a sketch of a contour map with an appropriate contour interval.

(a)  $f(x, y) = 3x^2 - y^2$

(e)  $f(x, y) = \sin(x^2 + y^2)$

(b)  $f(x, y) = \frac{y}{x^2}$

(f)  $f(x, y, z) = 2x - 3y + 4z - 5$

(c)  $f(x, y) = x^4$

(g)  $f(x, y, z) = x^2 + y^2 - z$

(d)  $f(x, y) = e^{x^2+4y^2}$

(h)  $f(x, y, z) = x^2 - y^2 + z^2$

2. Sketch a graph of the function  $f(x, y) = \sqrt{9 - x^2 - y^2}$ .

3. Evaluate the limit or determine it does not exist. You must justify your answer.

(a)  $\lim_{(x,y) \rightarrow (1,1)} \left( \frac{e^x - e^{-y}}{x + y} \right)$

(d)  $\lim_{(x,y) \rightarrow (0,0)} \left( \frac{x^3 + y^3}{xy^2} \right)$

(b)  $\lim_{(x,y) \rightarrow (1,0)} \ln(x^2 - y)$

(e)  $\lim_{(x,y) \rightarrow (0,0)} \left( \frac{x^3}{x^2 + y^2} \right)$

(c)  $\lim_{(x,y) \rightarrow (0,0)} \left( \frac{xy}{x^2 + y^2} \right)$

(f)  $\lim_{(x,y) \rightarrow (0,0)} \left( \tan(x) \sin \left( \frac{1}{|x| + |y|} \right) \right)$

4. Suppose two distinct contour curves of the function  $f(x, y)$  are tangent to each other at the point  $P = (a, b)$ . What can you say about  $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$ ? Explain your answer.