Difficulty guide for worksheet:	
C-level or B-level exam problem:	1a, 1b, 1c, 3, 4, 6
A-level exam problem or challenge for extra study:	1d, 1e, 2, 5
beyond the scope and/or removed from syllabus:	none

1. Find an equation of the plane $\mathcal{P}$ that satisfies the given conditions.

(a) $\mathcal{P}$ contains the point $(-1, 2, 9)$ and the vector $\boldsymbol{n} = \langle 9, 2, -1\rangle$ is normal to $\mathcal{P}$.

(b) $\mathcal{P}$ is parallel to the yz-plane and contains the point $(-2, 1, 3)$.

(c) $\mathcal{P}$ contains the point $(5, -6, 7)$ and is parallel to the plane $2x - y - 2 = 3z$.

(d) $\mathcal{P}$ contains the points $(5, 1, 1)$, $(1, 1, 2)$, and $(3, 1, -2)$.

(e) $\mathcal{P}$ passes through the origin and contains the line parametrized by $\boldsymbol{r}(t) = \langle 2 + t, 3 - 4t, t\rangle$. (Note that the line does not pass through the origin.)

2. For each part, determine whether the point lies on the line. If it does not, find an equation of the plane containing the line and the point. (Why can you find such a plane only in the negative case?)

(a) $(-7, 10, -3)$ and $\boldsymbol{r}(t) = \langle 1 - 4t, 6t - 5, t - 5\rangle$

(b) $(-1, 5, 9)$ and $\boldsymbol{r}(t) = \langle 4t + 3, t + 6, 5 - 4t\rangle$

3. Find the values of λ and μ so that the line parametrized by $\boldsymbol{r}(t) = \langle t+1, 2t+3, \lambda t+\mu\rangle$ is contained in the plane $2x - 3y + z = 9$.

4. Consider the following two parametrized lines.

$$\boldsymbol{r}_1(t) = \langle 2 + 2t,\ 8 + t,\ 10 + 3t\rangle$$
$$\boldsymbol{r}_2(t) = \langle 6 + t,\ 10 - 2t,\ 16 - t\rangle$$

(a) Determine whether the two lines intersect. If so, find the coordinates of that point.

(b) Suppose $\boldsymbol{r}_1(t)$ and $\boldsymbol{r}_2(t)$ describe the paths of two particles at time t. Do the particles collide? If so, at what time?

5. Consider the following two parametrized lines.

$$\boldsymbol{r}_1(t) = \langle 2t + 1,\ t - 4,\ 3t\rangle$$
$$\boldsymbol{r}_2(t) = \langle -t,\ -t + 1,\ t + \mu\rangle$$

(a) Find the value of μ so the two lines intersect in a single point, and find that intersection point.

(b) With the value of μ from part (a), find an equation of the plane containing both lines.

6. Find a parametrization of the line of intersection of the planes $-x + 2y + z = 1$ and $x + y + z = 0$.

Solutions

① (a) $9(x+1)+2(y-2)-(z-9)=0$

(b) $x=-2$

(c) $2(x-5)-(y+6)-2=3(z-7)$

(d) $\vec{u}=\langle -4,0,1\rangle$

$\vec{v}=\langle -2,0,-3\rangle$

$\vec{n}=\vec{u}\times\vec{v}=\langle 0,-14,0\rangle$

eq. of plane: $-14(y-1)=0$

OR $y=1$

(e) $\vec{u}=\vec{r}_1(0)=\langle 2,3,0\rangle$

$\vec{v}=\langle 1,-4,1\rangle$

$\vec{n}=\vec{u}\times\vec{v}=\langle 3,-2,-11\rangle$

eq. of plane: $3x-2y-11z=0$

② (a) $\left.\begin{array}{l} -7=1-4t \\ 10=6t-5 \\ -3=t-5 \end{array}\right\} \Rightarrow$ no solution

$\vec{u} = \vec{r}(0) - \langle -7, 10, -3 \rangle = \langle 8, -15, -2 \rangle$

$\vec{v} = \langle -4, 6, 1 \rangle$

$\vec{n} = \vec{u} \times \vec{v} = \langle -3, 0, -12 \rangle$

eq. of plane: $-3(x+7) - 12(z+3) = 0$

(b) $\left.\begin{aligned} -1 &= 4t+3 \\ 5 &= t+6 \\ 9 &= 5-4t \end{aligned}\right\} \Longrightarrow t = -1$

So point is on line; no unique plane containing one given line.

3 $2(t+1) - 3(2t+3) + (\lambda t + \mu) = 9$

$(-4+\lambda)t + (-16+\mu) = 0$

Must be true for all t, so $\lambda = 4$, $\mu = 16$.

4 (a) $\left.\begin{aligned} 2+2t &= 6+s \\ 8+t &= 10-2s \\ 10+3t &= 16-s \end{aligned}\right\} \Longrightarrow \begin{aligned} s &= -6 \\ t &= 4 \end{aligned}$

int. point: $\vec{r}_1(4) = \vec{r}_2(-6) = \langle 10, 12, 22 \rangle$

(b) No.

5 (a) $\left.\begin{aligned} 2t+1 &= -s \\ t-4 &= -s+1 \end{aligned}\right\} \Rightarrow s = 11, \ t = -6$

$3t = s + \mu \Rightarrow \mu = -29$

int. point: $\vec{r}_1(-6) = \langle -11, -10, -18 \rangle$

(b) $\vec{u} = \langle 2, 1, 3 \rangle$

$\vec{v} = \langle -1, -1, 1 \rangle$

$\vec{n} = \vec{u} \times \vec{v} = \langle 4, -5, -1 \rangle$

eq. of plane:

$4(x+11) - 5(y+10) - (z+18) = 0$

6 $\vec{n_1} = \langle -1, 2, 1 \rangle$

$\vec{n_2} = \langle 1, 1, 1 \rangle$

$\vec{v} = \vec{n_1} \times \vec{n_2} = \langle 1, 2, -3 \rangle$

Put $z=0$: $\left\{\begin{aligned} -x + 2y &= 1 \\ x + y &= 0 \end{aligned}\right\} \Rightarrow x = -\frac{1}{3}, y = \frac{1}{3}$

eq. of plane: $(x+\frac{1}{3}) + 2(y-\frac{1}{3}) - 3z = 0$