This worksheet serves as a review of only the truly essential material from Math 152 that you must have mastered to succeed in Math 251. Of course, you are expected to have mastered all of precalculus and algebra as well.

Methods of Integration

1. Find the following antiderivatives or integrals.

(a) $\displaystyle\int \frac{x^5}{\sqrt{16+x^2}}\,dx$

(b) $\displaystyle\int_1^{e^4} x^2\ln(x)\,dx$

(c) $\displaystyle\int \frac{1+2x^2}{(1-x^2)^{5/2}}\,dx$

(d) $\displaystyle\int_{-1}^{\sqrt{3}} \tan^{-1}(x)\,dx$

(e) $\displaystyle\int \frac{x^2-2x+8}{(x-2)(x^2+4)}\,dx$

(f) $\displaystyle\int \cosh(x)^3\sinh(x)^{17}\,dx$

Parametric Curves

2. Find a parametrization of each described curve.

(a) circle of radius 2 with center $(-1,3)$

(b) the ellipse $2x^2+3y^2=1$

(c) the line $y=3x+5$

(d) the line $x=-2$

3. For each described scenario, find a parametrization of the particle's path in the xy-plane.

(a) A particle starts at the point $(2,0)$ and then travels exactly once around the circle $x^2+y^2=4$ in the anticlockwise direction. (The particle does return to its starting point.)

(b) A particle starts at the point $(2,0)$ and then travels exactly three times around the circle $x^2+y^2=4$ in the anticlockwise direction. (The particle does return to its starting point.)

(c) A particle starts at the point $(0,2)$ and then travels exactly once around the circle $x^2+y^2=4$ in the anticlockwise direction. (The particle does return to its starting point.)

4. Find the length of the curve described by the parametrization $x(t)=e^{-2t}\cos(t)$ and $y(t)=e^{-2t}\sin(t)$ for $0\le t\le 2\pi$.

Polar Coordinates

5. Convert the given coordinates from rectangular to polar coordinates.

(a) $(1,0)$ (b) $(1,-\sqrt{3})$ (c) $(3,\sqrt{3})$ (d) $(-2,2)$

6. Sketch a graph of the equation $r=8\sin(4\theta)$ in the xy-plane. Then find the total area enclosed by the graph.

Solutions:

① (a) $\frac{1}{15}(16+x^2)^{1/2}(2048-64x^2+3x^4)+C$

(b) $\frac{1}{9}(1+11e^{12})$

(c) $\dfrac{x}{(1-x^2)^{3/2}}+C$

(d) $\left(\frac{1}{\sqrt{3}}-\frac{1}{4}\right)\pi-\frac{1}{2}\ln(2)$

(e) $\ln|x-2|-\tan^{-1}\left(\frac{x}{2}\right)+C$

(f) $\frac{1}{18}\sinh(x)^{18}+\frac{1}{20}\sinh(x)^{20}+C$

② (a) $x=-1+2\cos(t)$ $\quad 0\le t<2\pi$

$y=3+2\sin(t)$

(b) $x=\frac{1}{\sqrt{2}}\cos(t)$ $\quad 0\le t<2\pi$

$y=\frac{1}{\sqrt{3}}\sin(t)$

(c) $x=t$ $\quad t\in\mathbb{R}$

$$y = 3t + 5$$

(d) $x = -2$ $\quad t \in \mathbb{R}$

$y = t$

3 (a) $x = 2\cos(t)$ $\quad 0 \le t \le 2\pi$

$y = 2\sin(t)$

(b) $x = 2\cos(t)$ $\quad 0 \le t \le 6\pi$

$y = 2\sin(t)$

(c) $x = 2\cos(t + \pi/2)$ $\quad 0 \le t \le 2\pi$

$y = 2\sin(t + \pi/2)$

4 $$\int_0^{2\pi} \sqrt{5}\, e^{-2t}\, dt = \frac{\sqrt{5}}{2}\left(1 - e^{-4\pi}\right)$$

5 (a) $r = 1$, $\theta = 0$

(b) $r = 2$, $\theta = -\pi/3$

(c) $r = \sqrt{12}$ $= \pi/6$

(d) $r = \sqrt{8}$, $\theta = 3\pi/4$

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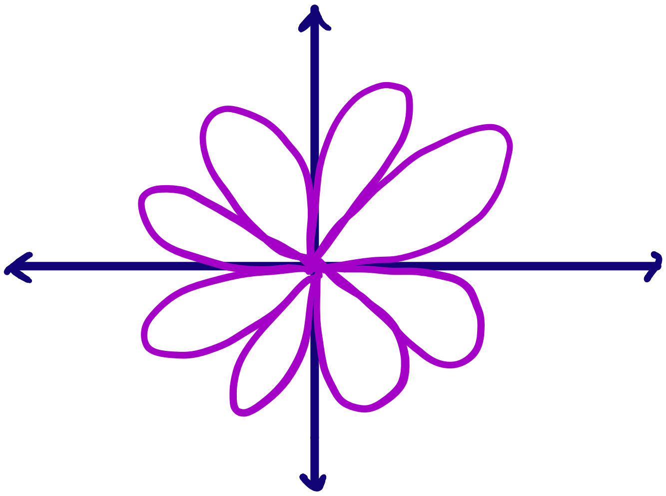

$$A = \frac{1}{2}\int_0^{2\pi} (8\sin(4\theta))^2 \, d\theta = 32\pi$$