Calculus 251:C3 Reading Guide - 6/16/2020

Section 14.6 Tangent Planes and Differentials

In §14.3, we saw what it means for a multivariable function to be differentiable. You should already know that single variable functions fail to be differentiable at discontinuities (of any type), cusps, corners, and vertical tangents. As you have probably already guessed, there is a lot more going on here even with only two variables. Of particular note is that in order for $f(x, y)$ to be differentiable at a point (a, b), it is *necessary* that $f_x(a, b)$ and $f_y(a, b)$ both exist, but that condition is not *sufficient*.

The intuition for single-variable calculus is that if you zoom in far enough on the graph of a differentiable function, the graph looks like a line. We used that fact to create linearizations of differentiable functions at particular points which we could use to estimate the function at x-values near the point. Remember estimating $\sqrt{4.02}$ without a calculator in calc I? For a function of two variables, the intuition is that as you zoom in on a differentiable function the graph looks more and more like a plane. The linearization at a point on the surface $z = f(x, y)$ is going to be a plane tangent to the surface at that point.

Find the following definitions/concepts/formulas/theorems:

- tangent plane to a level surface (definition and formula)
- normal line of a surface at a point (definition and formula)
- tangent plane to a surface
- formula for estimating change in value of differentiable multivariable function
- linearization of $f(x, y)$ centered at (x_0, y_0)
- Error in the standard linear approximation - You should look at this, but I can't imagine asking about it on a quiz or exam. It might(?) show up on a homework question. Ask me in office hours if you are curious!
- total differential (this should be familiar from calc 1, and is really just a sometimes convenient alternate notation)
- Extensions of the above concepts to functions of three variables

Examples 1-3 are all straightforward, and you will hopefully not have trouble working through them. Finding tangent planes uses techniques that we developed in chapter 12. Remember the examples where you had to find the plane containing two lines? These examples are why that was so important.

Examples 4-7 all involve using partial derivatives to produce lineariazations, and then use those linearizations to estimate the function somewhere near the point at which the linearization is centered. Example 4 doesn't produce an equation for the linearization, using the directional derivative to accomplish the same purpose. This works because we knew the direction of the motion in the domain whose effect we were going to be estimating. The other examples are more typical, where you find the linearization (possibly in differential form) and then use it to estimate function value changes.

Example 8 is a linearization of a three variable function. Is the linearization still a plane, or is it something else? If you graph $w = f(x, y, z)$, how many dimensions does the graph have?